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A LAYERWISE FINITE ELEMENT FORMULATION FOR THE FREE VIBRATION ANALYSIS OF LAMINATED COMPOSITE AND SANDWICH PLATES

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ABSTRACT:

A Layerwise Finite Element Formulation for the free vibration analysis of laminated composite and sandwich plates is an iterative numerical technique that simulates the behavior and performance of laminates and sandwich plates subjected to vibrational loads. To calculate the free vibration behavior of such laminated plates, the Layerwise Finite Element Formulation involves formulating a set of equations based on the nth layer of the laminates or multi-layered sandwich structure. The equations are then solved through assumed strain energy release methods and modal analysis, to determine the dispersion of vibrational energy across the laminates.

INTRODUCTION

In addition to the assumed strain energy method, the Layerwise Finite Element Formulation also utilizes the Finite Element Method (FEM) to determine the natural frequencies and mode shapes of a laminated or sandwich plate. The FEM is used to discretize and decompose the composite plate into several layers and finite elements, each of which contains strain energy and is affected by the distributed load. By calculating the strain energy of each element and element-to-element interaction, the modal shapes and natural frequencies of laminated or sandwich plates can be determined.[1]

The Layerwise Finite Element Formulation is a highly efficient and reliable method of solving free vibration analysis problems, as it is able to generate accurate and meaningful results in a fraction of the time it would take to solve the problem using traditional methods. Furthermore, the Layerwise Finite Element Formulation offers the ability to accurately model the effects of material nonlinearities such as anisotropic stiffness, kinematic nonlinearities such as Poisson's ratio, and defect induced nonlinearities such as shear strength. Finally, the Layerwise Finite Element Formulation is a relatively inexpensive and versatile numerical technique that can be used to simulate various types of vibrational loads.

Due to their low weight, high stiffness and high strength properties, the composites sandwich structures are widely used in various industrial areas e.g. civil constructions, marine industry, automobile and aerospace applications. A sandwich is a three layered construction, where a low weight thick core layer (e.g., rigid polyurethane foam) of adequate transverse shear rigidity, is sandwiched between two thin laminated composite face layers of higher rigidity (Pal and Niyogi 2009). Despite the many advantages of sandwich structures, their behavior becomes very complex due to the large variation of rigidity and material properties between the core and the face sheets. Therefore, the accuracy of the results for sandwich structures largely depends on the computational model adopted (Pandey and Pradyumna 2015).

In the literature, several two-dimensional theories and approaches have been used to study the behavior of composite sandwich structures. Starting by the simple classical laminated plate theory (CLPT), based on the Kirchhoff's assumptions, which does not includes the effect of the transverse shear deformation, the first-order shear deformation theory (FSDT), where the effect of the transverse shear deformation is considered (Reissner 1975, Whitney and Pagano 1970, Mindlin 1951, Yang et al. 1966), but this theory gives a state of constant shear stresses through the plate thickness, and the higher-order shear deformation theories (HSDT) where a better representation of transverse shear effect can be obtained (Lo et al. 1977, Manjunatha and Kant 1993, Reddy 1984, Lee and Kim 2013). Regarding the approaches used to model the behavior of composite structures, we distinguish the equivalent single layers (ESL) approach where all the laminate layers are referred to the same degrees-of-freedom (DOFs). The main advantages of ESL models are their inherent simplicity and their low computational cost, due to the small number of dependent variables. However, ESL approach fails to capture precisely the local behavior of sandwich structures. This drawback in ESL was circumvented by the Zig-Zag (ZZ) and layerwise (LW) approaches in which the variables are linked to specific layers (Belarbi and Tati 2015, Belarbi et al. 2016, Ćetković and Vuksanović 2009, Chakrabarti and Sheikh 2005, Kapuria and Nath 2013, Khalili et al. 2014, Khandelwal et al. 2013, Marjanović and Vuksanović 2014, Maturi et al. 2014, Sahoo and Singh 2014, Singh et al. 2011, Thai et al. 2016). For more details, the reader may refer to (Carrera 2002, Ha 1990, Khandan et al. 2012, Sayyad and Ghugal 2015).

In the last decades, the finite element method (FEM) has become established as a powerful method and as the most widely used method to analyze the complex behavior of composite sandwich structures (e.g., bending, vibration, buckling). This is due to the limitations in the analytical methods which are applicable only for certain geometry and boundary conditions (Kant and Swaminathan 2001, Mantari and Ore 2015, Maturi et al. 2014, Noor 1973, Plagianakos and Papadopoulos 2015, Srinivas and Rao 1970). Khatua and Cheung (1973) were one of the first to use the FEM in the analysis of this type of structures. They developed two triangular and rectangular elements to study the bending and free vibration of sandwich plates. In the recent years, many researchers have investigated the dynamic response of laminated com- posite and sandwich plates using finite element models based on Zig-Zag theory. Chakrabarti and Sheikh (2004) developed a C1 continuous six-noded triangular plate element with 48 DOFs for dy- namic analyses of laminate-faced sandwich plate using higher-order zig-zag theory (HZZT). After- wards, Kulkarni and Kapuria (2008) extended the application of a newly improved discrete Kirchoff quadrilateral element, based on third order zigzag theory to vibration analysis of composite and sandwich plates. Zhen and Wanji (2006, 2010) carried out free vibration analyses of laminated com- posite and sandwich plates using an eight-noded quadrilateral element based on a global-local higher order shear deformation theories (GLHSDT).

Chalak et al. (2013) developed a nine node finite element by taking out the nodal field variables in such a manner to overcome the problem of continuity requirement of the derivatives of transverse displacements for the free vibration analysis of laminated sandwich plate. An efficient nine-noded quadratic element with 99 DOFs is developed by Khandelwal et al. (2013) for accurately predicting natural frequencies of soft core sandwich plate. The formulation of this element is based on combined theory, HZZT and least square error (LSE) method. Therefore, the zig-zag plate theory presents good performance but it has a problem in its finite element implementation as it requires C1 continuity of transverse displacement at the nodes which involves finite element implantation difficulties. Also, it requires high-order derivatives for displacement when obtaining transverse shear stresses from equi- librium equations (Pandey and Pradyumna 2015).

Recently, various authors have adopted the layerwise approach to assume separate displacement field expansions within each material layer. Lee and Fan (1996) described a new layerwise model using the FSDT for the face sheets whereas the displacement field at the core is expressed in terms of the two face sheets displacements. They used a nine-nodded isoparametric finite element for bend- ing and free vibration of sandwich plates. On the other hand, a new three-dimensional (3D) layerwise finite element model with 240 DOFs has been developed by Nabarrete et al. (2003) for dynamic analyses of sandwich plates with laminated face sheets. They used the FSDT for the face sheets, whereas for the core a cubic and quadratic function for the in-plane and transverse, displacements, was adopted. In the same year, Desai et al. (2003) developed an eighteen-node layerwise mixed brick element with 108 DOFs for the free vibration analysis of multi-layered thick composite plates. Later, an eight nodes quadrilateral element having 136 DOFs was developed by Araújo et al. (2010) for the analysis of sandwich laminated plates with a viscoelastic core and laminated anisotropic face layers. The construction of this element is based on layerwise approach where the HSDT is used to model the core layer and the face sheets are modeled according to a FSDT. Elmalich and Rabinovitch (2012) have undertaken an analysis on the dynamics of sandwich plates, using a C0 four-node rectangular element. The formulation of this element is based on the use of a new layerwise model, where the FSDT is used for the face sheets and the HSDT is used for the core. In 2015, Pandey and Pradyumna (2015) presented a new higher-order layerwise plate formulation for static and free vibration analyses of laminated composite plates. A high-order displacement field is used for the middle layer and a first- order displacement field for top and bottom layers. The authors used an eight-noded isoparametric element containing 104 DOFs to model the plate. The performance of these layerwise models is good but it requires high computational effort as the number of variables dramatically increases with the number of layers.

According to the presented literature review on the sandwich models, we found that many authors used finite element models having large number of nodes and/or DOFs, especially those based on the layerwise approach. Therefore, the present work aims to propose a new C0 layerwise model competitor to the majority of aforementioned finite element models, having a reduced number of nodes and DOFs. This new model is used for the calculation of natural frequencies of laminated composite and sandwich plates. Thanks for enforcing the continuity of the interlaminar displacement, the number of variables is independent of the number of layers. The numerical results obtained by developed model are compared favorably with those obtained via analytical solution and numerical results ob- tained by other models. The results obtained from this investigation will be useful for a more under- standing of the bending and free vibration behavior of sandwich laminates plates.

MATHEMATICAL MODEL

Sandwich plate is a structure composed of three principal layers as shown in Figure.1, two face sheets (top-bottom) of thicknesses (ht), (hb) respectively, and a central layer named core of thickness (hc) which is thicker than the previous ones. Total thickness (h) of the plate is the sum of these thicknesses. The plane (x, y) coordinate system coincides with mid-plane plate.

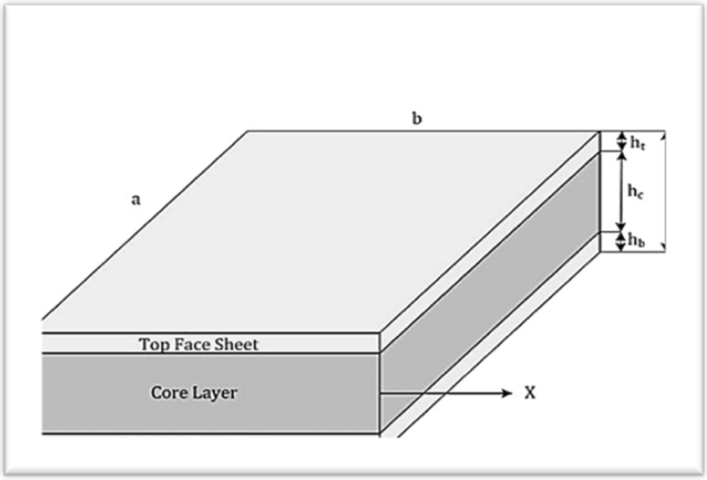

Figure 1: Geometry and notations of a sandwich plate.

2.1 Displacement Field

In the present model, the core layer is modeled using the HSDT. Hence, the displacement field is written as a third-order Taylor series expansion of the in-plane displacements in the thickness coor- dinate, and as a constant one for the transverse displacement:

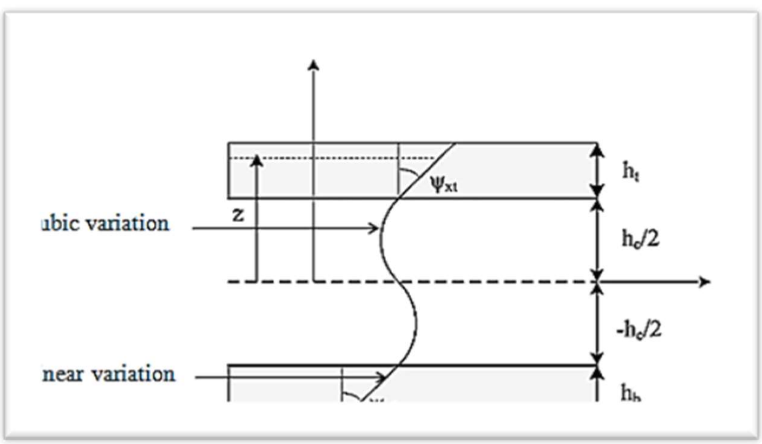

Figure 2: Representation of layerwise kinematics and coordinate system.

Traditional DOF for each face sheet, six rotational DOF for the core, while the three translations DOF are common for sandwich layers (Figure.3).

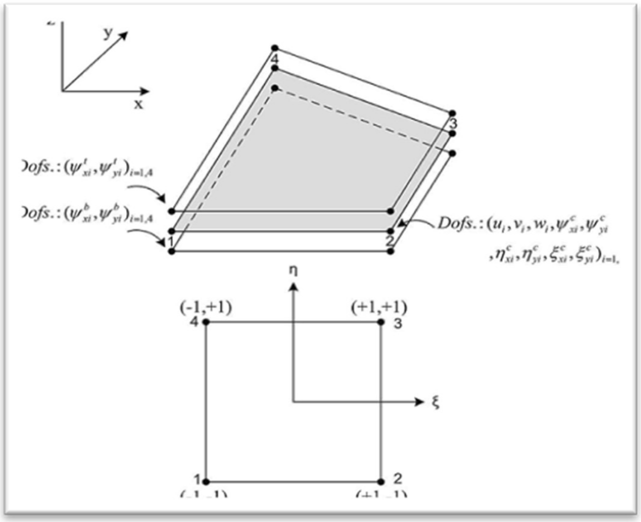

Figure 3: Geometry and corresponding DOFs of the present element.

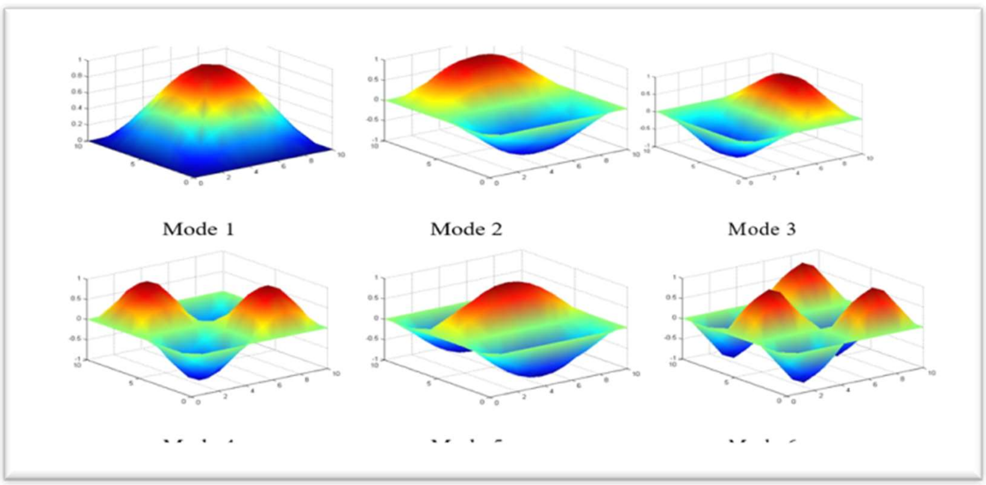

Figure 4: First six mode shapes of SSSS square laminated sandwich plate (0/90/C/90/0) with a/h =10.

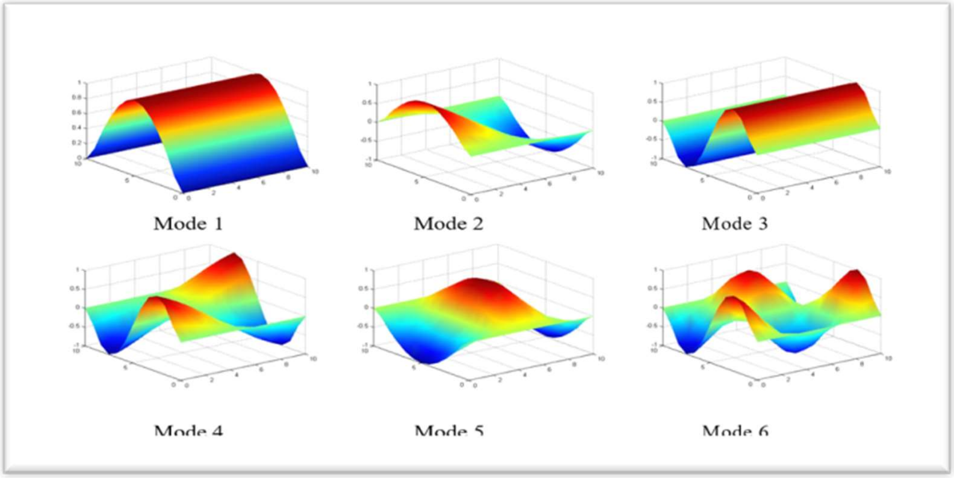

Figure 5: First six mode shapes of CFCF square laminated sandwich plate (0/90/C/90/0) with a/h =10.

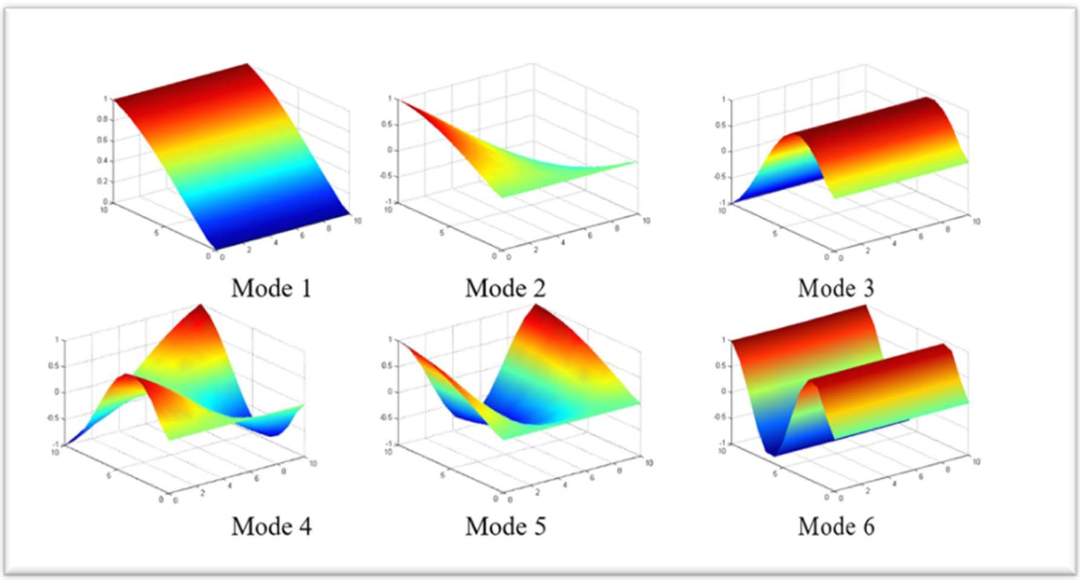

Figure 6: First six mode shapes of CFFF square laminated sandwich plate (0/90/C/90/0) with a/h =10.

Skew Laminated Plates

In order to evaluate the performance of the developed element for the study of free vibration response of irregular plates, a five layer symmetric cross-ply skew laminated plates (90/0/90/0/90) with simply supported edges is considered. The geometry of the skew plates is shown in Figure 7. The material properties MM5 of Table 2 is used for this analysis. The skew angle α is varied from 0°, 15°, 30°, 45° and 60°. The non-dimensional natural frequencies for the first four modes are reported in Table 6, considering the thickness ratios (a/h) as 10. A mesh size of 12×12 is considered for the analysis. The first six flexural mode shapes obtained for α = 45° are shown in Figure. 8. The comparison was made with the analytical solutions of Wang (1997) using B-spline Rayleigh-Ritz method, the solution of Ferreira et al. (2005) based on

Radial Basic Function (RBF), as well as with the finite element models of Nguyen-Van (2009) and Garg et al. (2006). The results of the comparison show the effectiveness of the present element in the analysis of this type of structures.

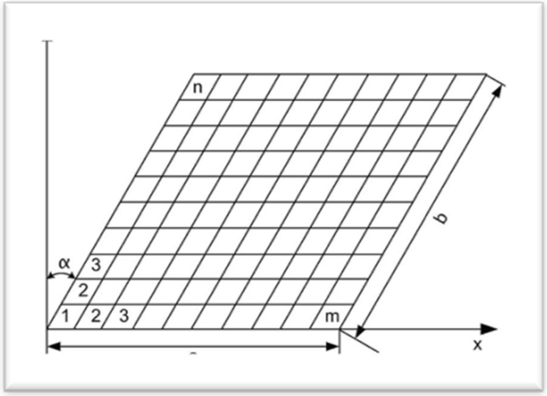

Figure 7: A skew plate with mesh arrangement (mesh size: m × n).

CONCLUSION:

A new higher-order layerwise finite element model was proposed for free vibration analysis of lami- nated composite and sandwich plates. The developed model is based on a proper combination of higher-order and first-order, shear deformation theories. These combined theories satisfy interlaminar displacement continuity. Although the model is a layerwise one, the number of variables is independ- ent of the number of layers. Thus, the plate theory enjoys the advantage of a single-layer plate theory, even though it is based on the concept of a layerwise plate approach. Based on this model, a four- noded C0 continuous isoparametric element is formulated. The performance and the efficiency of the newly developed FE model are demonstrated by several numerical examples on free vibration analysis of laminated composite, symmetric/unsymmetric sandwich and skew plates, with varying material combinations, aspect ratios, number of layers, geometry and boundary conditions. The results ob- tained by our model were compared with those obtained by the analytical results and other finite element models found in literature. The comparison showed that the element has an excellent accu- racy and a broad range of applicability. It is important to mention here, that the proposed FE for- mulation is simple and accurate in solving the free vibration problems of laminated composite and sandwich plates.

References

1. Araújo, A., Soares, C. M. & Soares, C. M. (2010), "Finite element model for hybrid active-passive damping analysis of anisotropic laminated sandwich structures", Journal of Sandwich Structures and Materials, 12 (4) 397-419

2. Belarbi, M. O. & Tati, A. (2015), "A new C0 finite element model for the analysis of sandwich plates using combined theories", International Journal of Structural Engineering, 6 (3) 212-239

3. Belarbi, M. O., Tati, A., Ounis, H. & Benchabane, A. (2016), "Development of a 2D isoparametric finite element model based on the layerwise approach for the bending analysis of sandwich plates", Structural Engineering and Mechanics, 57 (3) 473-506

4. Burlayenko, V., Altenbach, H. & Sadowski, T. (2015), "An evaluation of displacement-based finite element models used for free vibration analysis of homogeneous and composite plates", Journal of Sound and Vibration, 358 152-175

5. Carrera, E. (2002), "Theories and finite elements for multilayered, anisotropic, composite plates and shells", Archives of Computational Methods in Engineering, 9 (2) 87-140

6. Ćetković, M. & Vuksanović, D. (2009), "Bending, free vibrations and buckling of laminated composite and sandwich plates using a layerwise displacement model", Composite structures, 88 (2) 219-227

7. Chakrabarti, A. & Sheikh, A. H. (2004), "Vibration of laminate-faced sandwich plate by a new refined element", Journal of Aerospace Engineering, 17 (3) 123-134

8. Chakrabarti, A. & Sheikh, A. H. (2005), "Analysis of laminated sandwich plates based on interlaminar shear stress continuous plate theory", Journal of engineering mechanics, 131 (4) 377-384

9. Chalak, H. D., Chakrabarti, A., Iqbal, M. A. & Sheikh, A. H. (2013), "Free vibration analysis of laminated soft core sandwich plates", Journal of Vibration and Acoustics, 135 (1) 011013

10. Desai, Y. M., Ramtekkar, G. S. & Shah, A. H. (2003), "Dynamic analysis of laminated composite plates using a layer- wise mixed finite element model", Composite Structures, 59 (2) 237-249

11. Elmalich, D. & Rabinovitch, O. (2012), "A high-order finite element for dynamic analysis of soft-core sandwich plates", Journal of Sandwich Structures and Materials, 14 (5) 525-555