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W-PROJECTINE CURVATURE TENSOR OF LOCALLY CONFORMAL KAHLER MANIFOLD

Shahad Rifaat Khaleel [1], Dr. Prof. Ali Abdul Al Majeed Shihab [2]

[1,2] Department of Mathematics, College of Education for Pure Sciences, Tikrit University, Tikrit, Iraq.

[1] shahad.rf.iq2020@gmail.com, [2] ali.abd82@yahoo.com, draliabd@tu.edu.iq

Abstract

In this paper we stady the relationship between tensor is an algebraic curvature tensor , W- Projective Curvature Tensor of a Lokally Conformal Kahler manifold w_4., i . e. "it has a classical symmetry properties of the Riemann carvatur tensor" of a Lokally Conformal Kahler manifold w_4.has been examined in this research.

The typical Riemannian curvature symmetry features of this tensor were demonstrated. In the L.C.K- manifold, calculate the W- Projective tensor (W- tensor) components. Some observations and relationships among them were obtained, and links between the tensor components of this manifold were constructed. With obtaining a neutral equation for each of the eight these components.

Keywords: Locally conformal Kahler manifold w_4., W-projective tensor, conformal curvature tensor.

1.Introduction

Conformal transformations of Riemannian structures are the important object of differential geometry, where this "transformations which keeping the property of smooth harmonic function .It is" known, that such transformations have tensor in variant so-called W- Projective Curvature Tensor, In this paper we investigated the W- Projective Curvature Tensor of a Lokally Conformal Kahler manifold w4."

The W-projective curvature tensor: [2] on AH-manifold W is a tensor of type (4,0) and satisfied the relation $e^{-2f}W(A,B,C,D) = W(A,B,C,D)$, which is defined as the form:

$$W(A,B,C,D) = R(A,B,C,D) - \frac{1}{2n-1}[g(A,D)S(B,C) - g(B,D)S(A,C)]$$

Where R is the Riemannian curvature tensor. g is the Riemannian metric and A is the scalar curvature A,B,C,D $\in$ X(M). Where X(M) is the Lie algebra of W^∞ vector field on W.

The locally conformal Kahler manifold which are going to be dealt with in this study, is one of the sixteen classes of almost Hermitian manifold. The first study on locally conformal Kahler manifold was conducted by Libermann 1955 [3]. Vaisman, in 1981 put down some geometrical conditions for locally conformal Kahler manifold [9]. Letter on in1982, Tricerri mentioned different examples about the locally conformal Kahler manifold[8].

In 1993, Banaru [1] From the Banaru's classification of L.C.K-manifold. The class locally conformal kahler manifold statistics the following conditions:

$$B^{abc} = 0\,, B^{ab}_{c} = \alpha^{[a}\delta^{b]}_{c}$$

2.Preliminaries

Let W be a smooth 2n dimension manifold,$C^\infty(M)$- soft function algebra onW; α(M)vector fields of smoothness module on "manifold" of W; g = <.,.> - Riemannian metrics is a

Riemannian metrics link g onW, d: the element of distinction from the outside. The smooth class is assumed for all manifolds, Tensor fields, and other objecets C in the following. The structure of NK ("nearlykahler") on the "(manifold W)" is a pair "(Q,g)" where Q: represents the structure of the almost complicated ("$Q^2 = id$") onW, g=<.,.> represents the Riemannian "(pseudo)" metric onW, where in this case <Qα,Qβ> =<α, β> ; α, $\beta \in \alpha$(M).
Let W be a n1-dimensional 2n-dimensional smooth manifold. On W, C(M) is a smooth function algebra., and α(M) is the vector field module on W. The Riemannian conection of the metric is denoted by g, while the exterior differentiation is denoted by d.

3. The Structure Equation Of Locally Conformal Kahler Manifold

Definition 3.1:[4]

The W-projective curvature tensor on NK-manifold M is a tensor of type (4, 0) and satisfied the relation $e^{-2f}W(A,B,C,D) = W(A,B,C,D)$ which is defined as the form:

$$W(A,B,C,D) = R(A,B,C,D) - \frac{1}{2n-1}[g(A,D)S(B,C) - g(B,D)S(A,C)]$$

Where R is the Riemannian curvature tensor. g is the Riemannian metric and A is the scalar curvature A,B,C,D $\in$ X(M). Where X(M) is the Lie algebra of W^∞ vector field on M.

Definition 3.2:[4]

The W-projective curvature tensor on LCK-manifold M is a tensor of type (4, 0) and satisfied the relation $e^{-2f}W(A,B,C,D) = W(A,B,C,D)$ which is defined as the form:

$$W(A,B,C,D) = R(A,B,C,D) - \frac{1}{2n-1}[g(A,D)S(B,C) - g(B,D)S(A,C)]$$

Where R is the Riemannian curvature tensor. g is the Riemannian metric and A is the scalar curvature A,B,C,D $\in$ X(M). Where X(M) is the Lie algebra of W^∞ vector field on M.

Definition 3.3:[4]

The from defines the W-projective curvature tensor $W_{ijkI} = R_{ijkI} - \frac{1}{2n-1}[S_{jk}g_{iI} - S_{ik}g_{iI}]$

The Riemannian curvature tensor and the Ricci tensor, respectively are R and S.

Definition 3.4:[5]

A Riemannian manifold is called and Einstein manifold if the component of Ricci tensor satisfies the equation $r_{ij} = eg_{ij}$, where e and g are respectively an Einstein constant and Riemannian metric let's consider properties of the W-projective curvature tensor.

Remark 3.5: [1]

1- From the Banaru's classification of L.C.K-manifold. The class locally conformal kahler manifold statistics the following conditions:

$$B^{abc} = 0\,, B^{ab}_{c} = \alpha^{[a}\delta^{b]}_{c}$$

2- The value of Riemannian metric is g denoted by the form

i- $g_{ab} = g_{\hat{a}\hat{b}} = 0$

ii- $g_{\hat{a}b} = \delta^a_b$

iii- $g_{a\hat{b}} = \delta^b_a$

The structure equation of L.C.K-manifold provide by the following theorem.

Theorem 3.6: [6]

The component of the Riemannian curvature tensor of L.C.K-manifold in the ad joint G-structure space are given as the following forms:

i- $R_{abcd} = 0$

ii- $R_{\hat{a}\hat{b}\hat{c}\hat{d}} = 0$

iii- $R_{\hat{a}bcd} = \alpha_{a[c}\delta^{b}_{d]} + \frac{1}{2}\alpha_a\alpha_{[c}\delta^{b}_{d]}$

iv- $R_{a\hat{b}cd} = -\alpha_{a[c}\delta^{b}_{d]} - \frac{1}{2}\alpha_a\alpha_{[c}\delta^{b}_{d]}$

v- $R_{ab\hat{c}d} = \alpha_{[a|d|}\delta^{c}_{b]} - \alpha_{[a}\delta^{h}_{b]}\alpha_{[h}\delta^{c}_{d]}$

vi- $R_{abc\hat{d}} = \alpha_{[a|c|}\delta^{d}_{b]} - \alpha_{[a}\delta^{h}_{b]}\alpha_{[h}\delta^{d}_{c]}$

vii- $R_{\hat{a}\hat{b}cd} = -2\,\alpha^{[a}_{[c}\delta^{b]}_{d]}$

viii- $R_{ab\hat{c}\hat{d}} = 2\,\alpha^{[c}_{[a}\delta^{d]}_{b]}$

ix- $R_{\hat{a}b\hat{c}d} = A^{ac}_{bd} - \alpha^{[a}\delta^{h]}_{d}\alpha_{[b}\delta^{c}_{h]}$

x- $R_{\hat{a}bc\hat{d}} = A^{ad}_{bc} - \alpha^{[a}\delta^{h]}_{c}\alpha_{[h}\delta^{d}_{b]}$

xi- $R_{a\hat{b}\hat{c}d} = -A^{bc}_{ad} + \alpha^{[h}\delta^{b]}_{d}\alpha_{[a}\delta^{c}_{h]}$

xii- $R_{a\hat{b}c\hat{d}} = -A^{bd}_{ac} + \alpha^{[b}\delta^{h]}_{c}\alpha_{[a}\delta^{d}_{h]}$

xiii- $R_{\hat{a}\hat{b}\hat{c}d} = -\alpha^{[a|c|}\delta^{b]}_{a} + \alpha^{[a}\delta^{b]}_{h}\alpha^{[h}\delta^{c]}_{d}$

xiv- $R_{\hat{a}\hat{b}c\hat{d}} = -\alpha^{[a|d|}\delta^{b]}_{c} + \alpha^{[a}\delta^{b]}_{h}\alpha^{[h}\delta^{d]}_{c}$

xv- $R_{\hat{a}b\hat{c}\hat{d}} = \alpha^{a[c}\delta^{d]}_{b} + \frac{1}{2}\alpha^{a}\alpha^{[c}\delta^{d]}_{b}$

xvi- $R_{a\hat{b}\hat{c}\hat{d}} = -\alpha^{a[c}\delta^{d]}_{b} - \frac{1}{2}\alpha^{a}\alpha^{[c}\delta^{d]}_{b}$

Theorem 3.7:

The component of Ricci tensor of L.C.K-manifold in the ad joint G-structure space are given by the following forms:

i- $r_{ab} = \alpha c_{[b}\delta^{a}_{c]} + \frac{1}{2}\alpha c\alpha_{[b}\delta^{a}_{c]} + \alpha_{[c|b|}\delta^{c}_{a]} - \alpha_{[c}\delta^{h}_{a]}\alpha_{[h}\delta^{b}_{c]}$

ii- $r_{\hat{a}b} = -2\,\alpha^{[c}_{[b}\delta^{a]}_{c]} - A^{ac}_{cb} + \alpha^{[a}\delta^{h]}_{b}\alpha_{[c}\delta^{c}_{h]}$

iii- $r_{a\hat{b}} = A^{cb}_{ac} - \alpha^{[c}\delta^{h]}_{c}\alpha_{[a}\delta^{c}_{h]} + 2\,\alpha^{[b}_{[c}\delta^{c]}_{a]}$

iv- $r_{\hat{a}\hat{b}} = -\alpha^{[c|b|}\delta^{a]}_{c} + \alpha^{[c}\delta^{a]}_{h}\alpha^{[h}\delta^{b]}_{c} - \alpha^{c[b}\delta^{c]}_{a} - \frac{1}{2}\alpha^{c}\alpha^{[b}\delta^{c]}_{a}$

Theorem 3.8:

Thus, the projective tensor satisfies all of the algebraic curvature tensors characteristics

i. W(a, b, c, d) = - W(b, a, c, d)

ii. W(a, b, c, d) = - W(a, b, c, d)

iii. W((a, b, c, d) + W(b, c, a, d) + W(c, a, b, d))= 0

iv. W(a, b, c, d) = W(c, d, a,b), a, b, c, d $\in$ X(M)

Proof:

We shall prove

i. $W(a,b,c,d) = R(a,b,c,d) - \frac{1}{2n-1}[g(a,d)S(b,c) - g(b,d)S(a,c)]$

$$= -R(b,a,c,d) + \frac{1}{2n-1}[-g(a,d)S(b,c) - g(b,d)S(a,c)]$$
$$= -W(b,a,c,d)$$

Properties are similarly proved

ii. W(a, b, c, d) = - W(a, b, c, d)

iii. W((a, b, c, d) + W(b, c, a, d) + W(c, a, b, d))= 0

iv. W(a, b, c, d) = W(c, d, a, b)

Covariant projective tensor type (3,1) have form

$$\mathrm{W}(\mathrm{a},\mathrm{b})\mathrm{c} = \mathrm{R}(\mathrm{a},\mathrm{b})\mathrm{c} + \frac{1}{2n-1}\{< a,c > b - < b,c > a\}$$

Where R is the Riemannian curvature tensors and a is the scalar curvature a,b.c,d ∈ X(M)

By definition of spectrum tensor.

$$W(A,B)C = W_0(a,b)c + W_1(a,b)c + W_2(a,b)c + W_3(a,b)c + W_4(a,b)c + W_5(a,b)c + W_6(a,b)c + W_7(a,b)c; a,b,c \in X(M)$$

Tensor $W_0(a,b)c$as non-zero the component can have only components of the form:

$$\{W_0{}^a{}_{bcd}, W_0{}^{\hat{a}}{}_{\hat{b}\hat{c}\hat{d}}\} = \{W^a_{bcd}, W^{\hat{a}}_{\hat{b}\hat{c}\hat{d}}\}$$

Tensor $W_1(a,b)c$- components of the form

$$\{W_1{}^a{}_{bc\hat{d}}, W_1{}^{\hat{a}}{}_{\hat{b}\hat{c}d}\} = \{W^a_{bc\hat{d}}, W^{\hat{a}}_{\hat{b}\hat{c}d}\}$$

Tensor $W_2(a,b)c$- components of the form

$$\{W_2{}^a{}_{b\hat{c}d}, W_2{}^{\hat{a}}{}_{\hat{b}c\hat{d}}\} = \{W^a_{b\hat{c}d}, W^{\hat{a}}_{\hat{b}c\hat{d}}\}$$

Tensor $W_3(a,b)c$- components of the form

$$\{W_3{}^a{}_{b\hat{c}\hat{d}}, W_3{}^{\hat{a}}{}_{\hat{b}cd}\} = \{W^a_{b\hat{c}\hat{d}}, W^{\hat{a}}_{\hat{b}cd}\}$$

Tensor $W_4(a,b)c$- components of the form

$$\{W_4{}^a{}_{\hat{b}cd}, W_4{}^{\hat{a}}{}_{b\hat{c}\hat{d}}\} = \{W^a_{\hat{b}cd}, W^{\hat{a}}_{b\hat{c}\hat{d}}\}$$

Tensor $W_5(a,b)c$- components of the form

$$\{W_5{}^a{}_{\hat{b}c\hat{d}}, W_5{}^{\hat{a}}{}_{b\hat{c}d}\} = \{W^a_{\hat{b}c\hat{d}}, W^{\hat{a}}_{b\hat{c}d}\}$$

Tensor $W_6(a,b)c$- components of the form

$$\{W_6{}^a{}_{\hat{b}\hat{c}d}, W_6{}^{\hat{a}}{}_{bc\hat{d}}\} = \{W^a_{\hat{b}\hat{c}d}, W^{\hat{a}}_{bc\hat{d}}\}$$

Tensor $W_7(a,b)c$- components of the form

$$\{W_7{}^a{}_{\hat{b}\hat{c}\hat{d}}, W_7{}^{\hat{a}}{}_{abc}\} = \{W^a_{\hat{b}\hat{c}\hat{d}}, W^{\hat{a}}_{abc}\}$$

Tensor$W_0 = W_0(a,b)c, W_1 = W_1(a,b)c, \dots, W_7 = W_7(a,b)c$

The basic invariants projective L.C.K-manifold will be name.

Definition 3.9:

L.C.kahler manifold for which $W_i = 0$ is L.C.K almost hermitian manifold of class $W_i, i = 0,1,\dots,7$

Theorem 3.10:

1. Locally kahler manifold of class W_0 characterized by identity.

W$(a,b)c$ - W(a, Jb)Jc - W(Ja, b)Jc - W(Ja, Jb)c - JW(a, b)Jc - JW(a, Jb)c - JW(Ja, b)c + JW(Ja, Jb)Jc = 0 , a,b,c,d ∈ X(M)

2. Locally kahler manifold of class W_1 characterized by identity.

W(a, b)c + W(a, Jb)Jc - W(Ja, b)Jc + W(Ja, Jb)c + JW(a, b)Jc - JW(a, Jb)c - JW(Ja, b)c - JW(Ja, Jb)Jc = 0, a,b,c,d ∈ X(M)

3. Locally kahler manifold of class W_2 characterized by identity.

W(a, b)c - W(a, Jb)Jc + W(Ja, b)Jc + W(Ja, Jb)c - JW(a, b)Jc - JW(a, Jb)c + JW(Ja, b)c - JW(Ja, Jb)Jc = 0, a,b,c,d ∈ X(M)

4. Locally kahler manifold of class W_3 characterized by identity.

W(a, b)c + W(a, Jb)Jc + W(Ja, b)Jc - W(Ja, Jb)c - JW(a, b)Jc + JW(a, Jb)c + JW(Ja, b)c +JW(Ja, Jb)Jc = 0, a,b,c,d ∈ X(M)

5. Locally kahler manifold of class W_4 characterized by identity.

W(a, b)c + W(a, Jb)Jc + W(Ja, b)Jc - W(Ja, Jb)c + JW(a, b)Jc - JW(a, Jb)c - JW(Ja, b)c - JW(Ja, Jb)Jc = 0, a,b,c,d ∈ X(M)

6. Locally kahler manifold of class W_5 characterized by identity.

W(a, b)c - W(a, Jb)Jc + W(Ja, b)Jc + W(Ja, Jb)c + JW(a, b)Jc + JW(a, Jb)c - JW(Ja, b)c + JW(Ja, Jb)Jc = 0, a,b,c,d ∈ X(M)

7. Locally kahler manifold of class W_6 characterized by identity.

W(a, b)c + W(a, Jb)Jc - W(Ja, b)Jc + W(Ja, Jb)c + JW(a, b)Jc - JW(a, Jb)c + JW(Ja, b)c + JW(Ja, Jb)Jc = 0, a,b,c,d ∈ X(M)

8. Locally kahler manifold of class W_7 characterized by identity.

W(a, b)c - W(a, Jb)Jc - W(Ja, b)Jc - W(Ja, Jb)c + JW(a, b)Jc + JW(a, Jb)c + JW(Ja, b)c - JW(Ja, Jb)Jc = 0, a,b,c,d ∈ X(M)

Proof:

1. Let L.C.K-manifold of class W_0, the manifold of class W_0 characterized by a condition

$W_0{}^{a}{}_{bcd} = 0, or\ W^{a}_{bcd} = 0$

i.e. $[W(\epsilon_c, \epsilon_d)\epsilon_b]^a \epsilon_a$

As $\sigma - aprojector$ on $D_J^{\sqrt{-1}}$, that $\sigma o\{W(\sigma A, \sigma B)\sigma C\} = 0$

$$i.e\ (id - \sqrt{-1}J\{W(a - \sqrt{-1}Ja)(b - \sqrt{-1}Jb)(c - \sqrt{-1}Jc)\} = 0$$

Eliminating the brackets could be received :

i.e.

W(a, b)c – W(a, Jb) – W(Ja, b)Jc – W(Ja, Jb)c – JW(a, b)Jc – JW(Ja,b) + JW(Ja,Jb)Jc – $\sqrt{-1}$\{W(a,b)Jc + W(a,Jb)c + W(Ja,b)c – W(Ja,b)Jc\}– \{JW(a,b)c –JW(a,Jb)Jc – JW(Ja,b)Jc – JW(Ja,Jb)c\} = 0

i.e.

1. W(a,b)c – W(a,Jb)Jc – W(Ja, b)Jc – W(Ja,Jb)c – JW(a,b)Jc – JW(a,Jb)c – JW(Ja,b)c + JW(Ja,Jb)Jc = 0 … (1.1)

2. W(a,b)Jc + W(a,Jb)c+ W(Ja, b)c – W(Ja,Jb)Jc + JW(a,b)c – JW(a,Jb)Jc – JW(Ja,b)Jc - JW(Ja,Jb)Jc =0 …(1.2)

These equation (1) and (2) are interchangeable the first replacement yields, the second equality C and JC.

Thus (locally kahler)-identity characterizes a class W_0 manifold

W(a,b)c – W(a,Jb)Jc – W(Ja,b)Jc – W(Ja,Jb)c – JW(a,b)Jc – JW(a,Jb)c – JW(Ja,b)c + JW(Ja,Jb)Jc = 0 , a,b,c,d ∈ X(M)
Similarly considering locally kahler-manifold of classes $W_1 - W_7$ can be received the 2, 3, 4, 5, 6, 7 and 8.

Theorem 3.11:

We have the following inclusion relations:

1. $W_0 = W_3 = W_5 = W_6 = W_7$
2. $W_1 = -W_2$

Proof:

1. We shall prove $W_5 = W_6$and similarly. The other will be proven for an example, proving equality

Let (M,J,g) be L.C.K-manifold of class W_5 i.e $W^a_{\hat{b}c\hat{d}}$ then, according to (4) we have $W^a_{\hat{b}\hat{c}d}$=0, i.e. the AH-manifold is manifold of class W_6 back, let M-LCK-manifold of class W_6. Then$W^a_{\hat{b}\hat{c}d}$, so according to (4) and $W^a_{\hat{b}c\hat{d}} = 0$
Thus, classes W_5 and W_6 of L.C.K-manifold are coincide.

2. Prove inclusion $W_1 = -W_2$

Let (M,J,g) L.C.K-manifold of class W_2, i.e take place equality $W^a_{bc\hat{}} = W^a_{b\hat{d}c} = 0$. According to property (1.2) we have
$W^a_{\hat{b}cd} + W^a_{cd\hat{b}} + W^a_{d\hat{b}c} = 0$ i.e $W^a_{\hat{b}cd} = 0$ this the L.C.K-manifold of a class $W_1 = -W_2$ is L.C.K-manifold
Putting equality (2) and (3) we shall receive identity describing L.C.K-manifold of class $W_1 = -W_2$
W(A,B)C + W(JA,JB)C +JW(A,B)JC – JW(JA,JB)JC = 0, A,B,C,D ∈X(M)… (1.3)
From equality (1), (4), (6), (7) we shall receive the identity L.C.K-manifold of class $W_0 = W_3 = W_5 = W_6$
W(A,B)C +JW(JA,JB)JC = 0, A,B,C,D ∈X(M)… (1.4).

Theorem 3.12:

The following equations describes the components of the projective tensor of L.C.K-manifold in the adjoint G-structure

1. $W_{\hat{a}\hat{b}cd} = 2B^{abh}B_{dch} - \frac{1}{2n-1}[(3B^{abh}B_{cha} - A^{ba}_{ac})\delta^a_d - (3B^{bah}B_{hcb} - A^{ab}_{bc})\delta^b_d]$
2. $W_{\hat{a}b\hat{c}d} = B^{adc}B_{bdh} - A^{ac}_{bd} - \frac{1}{2n-1}(3B_{ab}\ B^{ach} - A^{ab}_{bd})\delta^a_d$
3. $W_{\hat{a}bc\hat{d}} = B^{adh}B_{hbc} + A^{ad}_{bc} + \frac{1}{2n-1}(3B^{ba}\ B_{chb} + A^{ab}_{bc})\delta^b_d$

And the others are either conjugate of above component or equal to zero.

Proof:

By using theorem3.6, we compute the components of projective tensor as the following:

1. $put\ i = a\ ,\quad j = b,\quad k = c,\quad I = d$

$$W_{abcd} = R_{abcd} - \frac{1}{2n-1}[g_{ad}S_{bc} - g_{bd}S_{ac}]$$

$$W_{abcd} = 0$$

2. $put\ i = \hat{a}\,,\quad j = b,\quad k = c,\quad I = d$

$$W_{\hat{a}bcd} = R_{\hat{a}bcd} - \frac{1}{2n-1}[g_{\hat{a}d}S_{bc} - g_{bd}S_{\hat{a}c}]$$

$$W_{\hat{a}bcd} = \alpha_{a[c}\delta^{b}_{d]} + \frac{1}{2}\alpha_a\alpha_{[c}\delta^{b}_{d]}$$

3. $put\ i = a\,,\quad j = \hat{b},\quad k = c,\quad I = d$

$$W_{a\hat{b}cd} = R_{a\hat{b}cd} - \frac{1}{2n-1}[g_{ad}S_{\hat{b}c} - g_{\hat{b}d}S_{ac}]$$

$$W_{a\hat{b}cd} = -\alpha_{a[c}\delta^{b}_{d]} - \frac{1}{2}\alpha_a\alpha_{[c}\delta^{b}_{d]}$$

4. $put\ i = a\,,\quad j = b,\quad k = \hat{c},\quad I = d$

$$W_{ab\hat{c}d} = R_{ab\hat{c}d} - \frac{1}{2n-1}[g_{ad}S_{b\hat{c}} - g_{bd}S_{a\hat{c}}]$$

$$W_{ab\hat{c}d} = \alpha_{[a|d|}\delta^{c}_{b]} - \alpha_{[a}\delta^{h}_{b]}\,\alpha_{[h}\delta^{c}_{d]}$$

5. $put\ i = a\,,\quad j = b,\quad k = c,\quad I = \hat{d}$

$$W_{abc\hat{d}} = R_{abc\hat{d}} - \frac{1}{2n-1}[g_{a\hat{d}}S_{bc} - g_{b\hat{d}}S_{ac}]$$

$$W_{abc\hat{d}} = \alpha_{[a|c|}\delta^{d}_{b]} - \alpha_{[a}\delta^{h}_{b]}\,\alpha_{[h}\delta^{d}_{c]}$$

6. $put\ i = \hat{a}\,,\quad j = \hat{b},\quad k = c,\quad I = d$

$$W_{\hat{a}\hat{b}cd} = R_{\hat{a}\hat{b}cd} - \frac{1}{2n-1}[g_{\hat{a}d}S_{\hat{b}c} - g_{\hat{b}d}S_{\hat{a}c}]$$

$$W_{abc\hat{d}} = -2\alpha^{[a}_{[c}\delta^{b]}_{d]} - \frac{1}{2n-1}[\left(-2\alpha^{[c}_{[b}\delta^{a]}_{c]} - A^{ac}_{cb} + \alpha^{[a}\delta^{h]}_{b}\,\alpha_{[c}\delta^{c}_{h]}\right)\delta^{a}_{b}$$

$$-\left(\left(-2\alpha^{[c}_{[b}\delta^{a]}_{c]} - A^{ac}_{cb} + \alpha^{[a}\delta^{h]}_{b}\,\alpha_{[c}\delta^{c}_{h]}\right)\delta^{a}_{b}\right)$$

$$= -2\alpha^{[a}_{[c}\delta^{b]}_{d]}$$

7. $put\ i = \hat{a}\,,\quad j = b,\quad k = \hat{c},\quad I = d$

$$W_{\hat{a}b\hat{c}d} = R_{\hat{a}b\hat{c}d} - \frac{1}{2n-1}[g_{\hat{a}d}S_{b\hat{c}} - g_{bd}S_{\hat{a}\hat{c}}]$$

$$W_{\hat{a}b\hat{c}d} = A^{ac}_{bd} - \alpha^{[a}\delta^{h]}_{d}\,\alpha_{[b}\delta^{c]}_{h]} - \frac{1}{2n-1}((A^{cb}_{ac} - \alpha^{[c}\delta^{h]}_{c}\,\alpha_{[a}\delta^{c}_{h]} + 2\alpha^{[b}_{[c}\delta^{c]}_{a]})\delta^{b}_{a}$$

8. $put\ i = \hat{a}\,,\quad j = b,\quad k = c,\quad I = \hat{d}$

$$W_{\hat{a}cb\hat{d}} = R_{\hat{a}cb\hat{d}} - \frac{1}{2n-1}[g_{\hat{a}\hat{d}}S_{bc} - g_{b\hat{d}}S_{\hat{a}c}]$$

$$W_{\hat{a}cb\hat{d}} = A^{ad}_{bc} - \alpha^{[a}\delta^{h]}_{c}\,\alpha_{[h}\delta^{d]}_{b]} - \frac{1}{2n-1} + (-2\alpha^{[c}_{[b}\delta^{a]}_{c]} - A^{ac}_{cb} + \alpha^{[a}\delta^{h]}_{b}\,\alpha_{[c}\delta^{c}_{h]})\delta^{b}_{a}$$

The above theorem calculated components projective tensor Curvature on space of adjoint G-structure projective tensor of L.C.K-manifold and W_1, W_2 and W_4 have only other components projective tensor that are equal to zero.

i.e. for L.C.K-manifold only three projective tensor don’ts equal zero $W_1$with component $\{W^{a}_{b\hat{c}d}, W^{\hat{a}}_{\hat{b}c\hat{d}}\}$ A Gray [2] defined three special classes of almost Hermitian manifold which are given in terms of Riemannian curvature tensor.

These classes are denoted by$R_1, R_2,$and R_3, class $R_1$is called a parakahler manifold [7]. The class R_3 is called R.K-manifold [10]. A Gray [1] proved that $R_1 \subset R_2 \subset R_3$, according to which key to understanding of differential geometrical properties kahler manifolds identities with which satisfies them Riemann curvature tensor are

$R_1: < R(A,B)C,D > = < R(JA,JB)C,D >$

$R_2: < R(A,B)C,D > =< R(JA,JB)C,D > + < R(JA,B)JC,D > +< R(JA,B)C,JD >$

$$R_3: < R(A,B)C,D > =< R(JA,JB)JC,JD >$$

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