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ON NANO GENERALIZED DELTA OPEN SET AND NANO GENERALIZED DELTA CONTINUOUS FUNCTION

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Abstract
The aim of this paper is to introduce a new class of sets called generalized delta open sets in topological spaces and some of their basic properties are investigated a new class of sets lies between g-δ-Reg-open set and g-δ-α-open set. We obtain their characterizations, their basic properties and their relationships with other types of sets in topological spaces. If provides definition of the new types of N-g-δ-continuous function which is called Ngδ(Reg, α, pre, semi, β) continuous functions to find the relationship with these types and necessary theorem and some examples.

Keywords: N-Reg-open (resp. N-α-open, N-semi-open, N-pre-open, N-β-open and N-δ-open) sets, N-Reg- continuous (resp. N-α- continuous, N-semi- continuous, N-pre- continuous, N-β- continuous and N-δ- continuous) functions.

1.Introduction:

In 2014, the concept of Nano generalized open(closed) sets introduced by Bhuvanswari[4]. Plays a significant role in general topology and may research papers were published which deal with different types of Nano generalized open(closed) sets. In 2014, Thanga[18] introduced the concept of N-g-α-closed(N-g-α-open)set. In 2014 Bhuvanswari[43] introduce Nano generalized pre closed set. These concepts motivated us to define a new class of set, called the Nano generalized delta open sets. Throughout this paper, $(U, T_R(X))$ or simply U represents Nano topological space, for a subset F of space $(U, T_R(X))$. The continuous function is concept in Nano generalized topology. In 2013, thivagar M.L[15] define a Nano continuous function. In 2015[2], Bhuvanswari define a Nano generalized continuous function. In 2015, Thivagar Mlellis [17] establish the notion of N-δ-open sets. And these concept have been massively investigated it was proved that N-δ-open set is (N-δ-pre and N-δ-semi) open set but the converse not be true. Noiri [9] introduce the notion of δ-continuous function and some theory.

2. Preliminaries:

Let us recall the following definitions which are useful in the :

Definitions 2.1: [17]

Let $(X, \mathcal{T})$ be a topological space and $F \subseteq X$, Then F is said to be:

1. δ-open if $F = int_\delta(F)$).[19]
2. Regular-open set $F = int(cl(F))$.[14]
3. α-open if $F \subseteq int(cl(int(F)))$.[8]
4. Pre-open if $F \subseteq int(cl(F))$.[6]
5. Semi-open if $F \subseteq cl(int(F))$.[5]
6. β-open if $F \subseteq cl(int(cl(F)))$.[1]

Definition 2.2 [16]:

Let U be a non-empty finite set of elements, called the universe and R be an equivalence relation on U named as the indiscernibility relation. The elements in the same class are said to be indiscernible with one another. The binary (U, R) is said to be the approximation space.

Let x $\subseteq$ U then,

1. The lower approximation of X with respect to R is a collection of all elements which can be classified as X with respect to R and symbolized by $L_R(x)$. That is $L_R(x) = \bigcup_{x\in U}\{R(x) \subseteq X\}$, where R(x) is the equivalence class determined by X.
2. The upper approximation of X with respect to R is a collection of all elements that is can be possibly classified as X with respect to R and symbolized by $U_R(x)$. that is $U_R(x) = \bigcup_{x\in U}\{R(x): R(x) \cap X \neq \phi\}$,
3. The boundary region of X with respect to R is a collection of all elements that is can be classified as X nor as not X with respect to R and symbolized by $B_R(x)$ that is $B_R(x) = U_R(x) - L_R(x)$.

Proposition 2.3 [16]:

If (U, R) is an approximation space and X, Y $\subseteq$ U, then:

1. $L_R(x) \cup L_R(y) \subseteq L_R(x \cup y)$.
2. $U_R(x \cup y) = U_R(x) \cup U_R(y)$
3. $U_R\, U_R(x) = L_R\, U_R(x) = U_R(x)$.
4. $L_R(x \cap y) = L_R(x) \cup L_R(y)$
5. $L_R\, L_R(x) = U_R\, L_R(x) = L_R(x)$
6. $U_R(x \cap y) \subseteq U_R(x) \cap U_R(y)$
7. $U_R(x^c) = [L_R(x)]^c$ and $L_R(x^c) = [U_R(x)]^c$
8. $L_R(x) \subseteq K \subseteq U_R(x)$
9. $L_R(x) \subseteq L_R(y)$ and $U_R(x) \subseteq U_R(y)$ where $x \subseteq y$.
10. $L_R(\emptyset) = U_R(\emptyset) = \emptyset$ and $L_R(U) = U_R(U) = U$.

Definition 2.4: [16]

Let U be the universe, R be an equivalence relation on U and $T_R(x) = \{U, \emptyset, B_R(x), L_R(x), U_R(x)\}$ satisfies the following axioms:

1. $\emptyset$ and $U \in T_R(x)$
2. The union of objects of any sub collection of $T_R(x)$ is in $T_R(x)$
3. The intersection of the objects of any sub collection of $T_R(x)$ is in $T_R(x)$.

Thus $T_R(x)$ is topology on U be called Nano-topology on U with respect to X. we call (U, $T_R(x)$) as the Nano-Topological space. The members of $T_R(x)$ is said to be Nano-open sets.

Definitions 2.5: [15]

Let $(U, T_R(X))$ be a Nano topological space and $F \subseteq U$ then F is said to be :

1. Nano δ-open set if $F = N\,int_\delta(F)$
2. Nano Reg open set if $F = Nint(Ncl(F))$.

3. Nano-α-open set if F⊆Nint(Ncl(Nint(F))).
4. Nano-pre-open set if F⊆Nint(Ncl(F)).
5. Nano-semi-open set if F⊆Ncl(Nint(F)).
6. Nano-β-open set if F⊆Ncl(Nint(Ncl(F))).

Definition 2.6:[17]

Let (U,T_R(X)) be a Nano topological space and F⊆U then, the Nδ-interior (resp. Nδ-closure) of F is denoted by $Nint_δ$(F) = ∪{D: D is NReg-open set and D ⊆ F} (resp. $Ncl_δ$(F) = ∪ {X ∈ U: Nint(Ncl(D)∩F ≠∅ and X ∈ D}).

Definitions 2.7: [17]

Let (U, T_R(X)) be a Nano topological space and F⊆U, then F is said to be:

1. N-δ-Reg-open if F = $Nint_δ$ ($Ncl_δ$(F)) .
2. N-δ-α-open if F ⊆ $Nint_δ$ ($Ncl_δ$($Nint_δ$ (F))).
3. N-δ-pre-open if F ⊆ $Nint_δ$ ($Ncl_δ$(F)).
4. N-δ-semi-open if F ⊆$Ncl_δ$($Nint_δ$ (F)).
5. N-δ-β-open if F ⊆ Ncl $_δ$($Nint_δ$($Ncl_δ$ (F))).

Definitions 2.8:

A function $f: (X, T) \to (Y, T)$is said to be:

1. Continuous if f^{-1} (D) is open set in X, ∀ open set D in Y.[9]
2. g-continuous if f^{-1} (D) is g-open set in X, ∀ open set D in Y.[14]
3. δ-continuous if f^{-1}(D) is δ-open set in X, ∀ open set D in Y.[9]
4. Reg-continuous if f^{-1}(D) is Reg-open set in X, ∀ open set D in Y.[9]
5. α-continuous if f^{-1}(D) is α-open set in X, ∀ open set D in Y.[9]
6. pre-continuous if f^{-1}(D) is pre-open set in X, ∀ open set D in Y.[9]
7. semi-continuous if f^{-1}(D) is semi-open set in X, ∀ open set D in Y.[9]
8. β-continuous if f^{-1}(D) is β-open set in X, ∀ open set D in Y.[9]

3.On Nano Generalized-δ-open set

Definitions 3.1:

Let (U, T_R(x)) be a Nano topological space, a subset F of (U, T_R(x)) is said to be:

1. Nano Generalized closed set (N-g-closed) if Ncl(F) ⊆ D where F⊆ D and D is N-open set in (U, T_R(x)). [4]
2. N-g-Reg-closed set if N-Reg-cl(F) ⊆ D where F ⊆ D, and D is N-open in (U, T_R(x)).[3]
3. N-g-α-closed set if N-α-cl(F) ⊆ D where F ⊆ D, and D is N-open in (U, T_R(x)).[3]
4. N-g-semi-closed set if N-semi-cl(F) ⊆ D where F ⊆ D, and D is N-open in (U, T_R(x)).[3]
5. N-g-pre-closed set if N-pre-cl(F) ⊆ D where F ⊆ D, and D is N-open in (U, T_R(x)).[3]
6. N-g-β-closed set if N-β-cl(F) ⊆ D where F ⊆ D, and D is N-open in (U, T_R(x))[3]

Definition 3.2:

Let (U, T_R(x)) be a Nano topological space, a subset F of (U, T_R(x)) is said to be N-g-δ-closed set if Nclδ (F) ⊆ D where F ⊆ D, and D is N-open in (U, T_R(x)).

Definition 3.3: [2]

Let (H, T_R(x)) be a Nano topological space and V ⊆ H then V is said to be N-g-open set(resp. N,-g-Reg-open , N-g-α-open, N-g-pre-open, N-g-semi-open and N-g-β-open) sets if its complement N-g-closed set(resp. N,-g-Reg-closed, N-g-α-closed, N-g-pre-closed, N-g-semi-closed and N-g-β-closed) sets.

Definition 3.4:

Let (H, T_R(x)) be a Nano topological space and V ⊆ H then V is said to be N-g-δ-open set if its complement is N-g-δ-closed set.

Definitions 3.5:

Let(U_1, T_R(x)) be a Nano topological space and F ⊆ U_1, then F is said to be:

1. N-g-δ-α-closed set if N-δ-α-cl(F) ⊆ D where F ⊆ D, and D is N-open in (U_1, T_R(x)).
2. N-g-δ-pre-closed set if N-δ-pre-cl(F) ⊆ D where F ⊆ D, and D is N-open in (U_1, T_R(x)).
3. N-g-δ-semi-closed set if N-δ-semi-cl(F) ⊆ D where F ⊆ D, and D is N-open in (U_1, T_R(x)).
4. N-g-δ-β-closed set if N-δ-β-cl(F) ⊆ D where F ⊆ D, and D is N-open in (U_1, T_R(x)).
5. N-g-δ-Reg-closed set if N-δ-Reg-cl(F) ⊆ D where F ⊆ D, and D is N-open in (U_1, T_R(x)).

Remark 3.6:

We can show the relation between some types of N-g-δ-closed set by the next diagram:

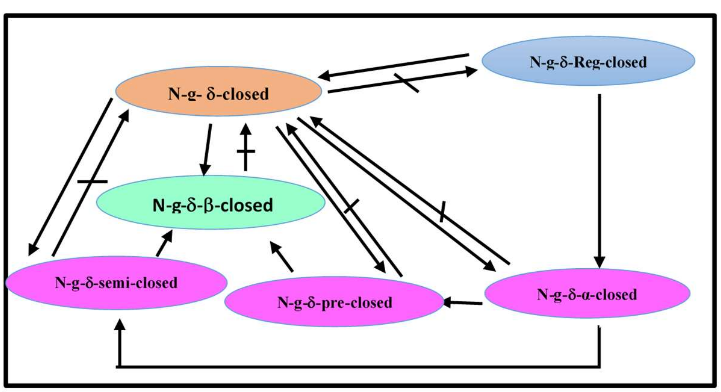

Diagram 3.1: the relation between some types of N-g-δ-closed sets

Definition 3.7:

Let (U_2, T_R(x)) be a Nano topological space and V ⊆ U_2 then V is said to be N-g-δ-α-open (resp. N-g-δ-pre-open, N-g-δ-semi-open, N-g-δ-β-open and N-g-δ-Reg-open) sets if its complement N-g-δ-α-closed(resp. N-g-δ-pre-closed, N-g-δ-semi-closed, N-g-δ-β-closed and N-g-δ-Reg-closed)) sets.

Remark 3.8:

We can show the relation between some type of N-g-δ-open set by the next diagram:

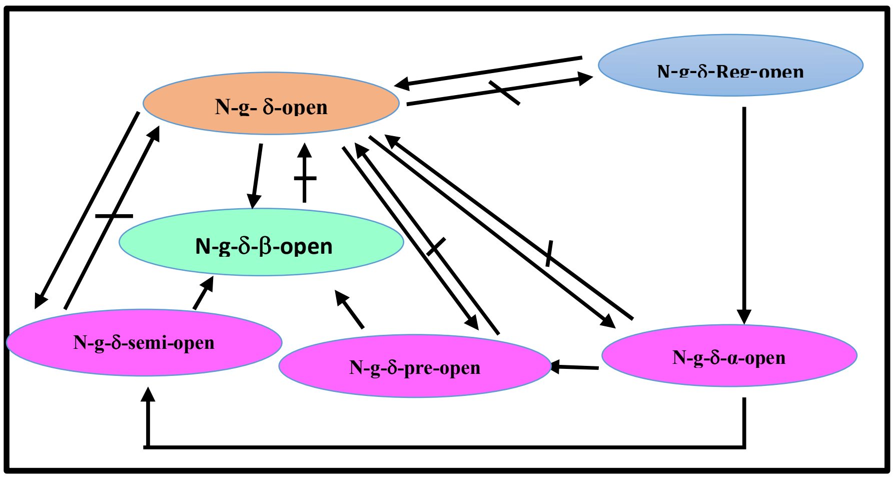

Diagram 3.2: the relation between some types of N-g-δ-open sets

Example 3.9:

Let U_1 = { 1, 2, 3, 4} with U_1/R = {{1}, {2, 4}} and let X = {1, 2} then T_R(x) = { U_1, ∅, {1}, {2, 4} {1, 2, 4}} and $T_R(x)^c$ = { U_1, ∅, {2, 3, 4}, {1,3}, {3}} then:

Table(3-1): Explain the relation between N-g-δ-closed sets

F	N-g-closed	N-g-δ closed	N-g-δ-Reg closed	N-g-δ-α-closed	N-g-δ-pre closed	N-g-δ-semi closed	N-g-δ-β closed
{1}	∅	∅	∅	∅	∅	{1}	{1}
{2}	∅	∅	∅	∅	{2}	{2}	{2}
{3}	{3}	{3}	{3}	{3}	{3}	{3}	{3}
{4}	∅	∅	∅	∅	{4}	{4}	{4}
{1,2}	∅	∅	∅	∅	∅	∅	∅
{1,3}	{1,3}	{1,3}	{1,3}	{1,3}	{1,3}	{1,3}	{1,3}
{1,4}	∅	∅	∅	∅	∅	∅	∅

{2,3}	{2,3}	{2,3}	{2,3}	{2,3}	{2,3}	{2,3}	{2,3}
{2,4}	∅	∅	∅	∅	∅	{2,4}	{2, 4}
{3,4}	{3,4}	{3,4}	{3,4}	{3,4}	{3,4}	{3,4}	{3,4}
{1,2,3}	{1,2,3}	{1,2,3}	{1,2,3}	{1,2,3}	{1,2,3}	{1,2,3}	{1,2,3}
{1,2,4}	∅	∅	∅	∅	∅	∅	∅
{1,3,4}	{1,3,4}	{1,3,4}	{1,3,4}	{1,3,4}	{1,3,4}	{1,3,4}	{1,3,4}
{2,3,4}	{2,3,4}	{2,3,4}	{2,3,4}	{2,3,4}	{2,3,4}	{2,3,4}	{2,3,4}
X	X	X	X	X	X	X	X
∅	∅	∅	∅	∅	∅	∅	∅

Example 3.10:

From **example 2.9**, we get

Table(3-2): Explain the relation between N-g-δ-open sets

F	N-g-open	N-g-δ open	N-g-δ-Reg open	N-g-δ-α-open	N-g-δ-pre open	N-g-δ-semi open	N-g-δ-β open
{1}	{1}	{1}	{1}	{1}	{1}	{1}	{1}
{2}	{2}	{2}	{2}	{2}	{2}	{2}	{2}
{3}	∅	∅	∅	∅	∅	∅	∅
{4}	{4}	{4}	{4}	{4}	{4}	{4}	{4}
{1,2}	{1,2}	{1,2}	{1,2}	{1,2}	{1,2}	{1,2}	{1,2}
{1,3}	∅	∅	∅	∅	∅	{1,3}	{1,3}
{1,4}	{1,4}	{1,4}	{1,4}	{1,4}	{1,4}	{1,4}	{1,4}
{2,3}	∅	∅	∅	∅	∅	∅	∅
{2,4}	{2,4}	{2,4}	{2,4}	{2,4}	{2,4}	{2,4}	{2,4}
{3,4}	∅	∅	∅	∅	∅	∅	∅
{1,2,3}	∅	∅	∅	∅	{1,2,3}	{1,2,3}	{1,2,3}
{1,2,4}	{1,2,4}	{1,2,4}	{1,2,4}	{1,2,4}	{1,2,4}	{1,2,4}	{1,2,4}
{1,3,4}	∅	∅	∅	∅	{1,3,4}	{1,3,4}	{1,3,4}
{2,3,4}	∅	∅	∅	∅	∅	{2,3,4}	{2,3,4}
X	X	X	X	X	X	X	X
∅	∅	∅	∅	∅	∅	∅	∅

Remarks 3.11:

1. Every N-g-δ-Reg-open set is N-g-δ-open but the converse not necessary true.
2. Every N-g-δ-open set is N-g-δ-α-open but the converse not necessary true.
3. Every N-g-δ-open set is N-g-δ-pre-open but the converse not necessary true. the set {1,2,3} is N-g-δ-pre-open set but not N-g-δ-open set.
4. Every N-g-δ-open set is N-g-δ-semi-open set but the converse not necessary true. The set {1,2,3} is N-g-δ-semi-open set but not N-g-δ-open set.

5. Every N-g-δ-open set is N-g-δ-β-open set but the converse not necessary true. The set {1,2,3} is N-g-δ-β-open set but not N-g-δ-open set.
6. Every N-δ-open set is N-g-δ-open set but the converse not necessary true. The set {2} is N-g-δ-open set but not N-δ-open set.

Theorem 3.12:

A subset F of a Nano topological space (U, $T_R(x)$) is N-g-δ-open set iff D ⊆Nintδ(F) wherever D is N-δ-closed and D⊆F.

Proof:

Let D be N-δ-closed set of (U, $T_R(x)$) and D ⊆ F. Then, (U-D) is N-δ-open set and (U-F) ⊆ (U-D). Since (U-F) is N-g-δ-closed set. Then Nclδ(U-F) ⊆ (U-D) which implies D ⊆Nintδ(F).

Conversely,

Let D_1 be a N-δ-open set of U and (U-F) ⊆ D_1. Since (U-D_1) is a N-δ-closed set contained in F. by hypothesis (U-D_1) ⊆Nintδ(F). That is U-Nintδ(F) = Nclδ(U-F) ⊆ D_1. Hence, U-F is N-g-δ-closed and so F is N-g-δ-open set.

Theorem 3.13:

F is N-g-δ-open set in the Nano topological space (U, $T_R(x)$) iff D= U_1 when D is N-δ-open set and Nint(F) ∪ F^c⊆ D.

Proof:

Let F is N-g-δ-open set and D is N-δ-open such that Nintδ(F) ∪F^c⊆ D, then D^c⊆F∩Nclδ(F^c) ⊆ Nclδ(F^c)-F^c. since F^c is N-g-δ-closed set, Nclδ(F^c) -F^c not contain any non-empty N-δ-closed set but D^c is N-δ-closed subset of Nclδ(F^c)-F^c. then D^c = ∅, that is D = U_1.

Conversely,

Where D is N-δ-open set and Nclδ(F) ∪ F^c ⊆ D. Then, D = U_1. Let F_1 be N-δ-closed set such that F_1⊆ F then Nintδ(F_1)∪$F_1{}^c$⊆Nintδ(F)∪F^c which is N-δ-open set, then Nintδ(F) ∪ F^c =U. that is F_1⊆ Nintδ(F). Since every x∈F_1 belong to Nintδ(F). Thus F_1⊆Nintδ(F). Where F_1 is N-δ-closed set and F_1⊆ F. then F is N-g-δ-open set.

4.On Nano Generalized-δ-Continuous function

Definitions 4.1:

Let (U_1, $T_R(x)$) and (U_2, $T_R(Y)$) be two Nano topological space. Then, a mapping f: (U_1, $T_R(x)$) →(U_2, $T_R(Y)$) is said to be:

1. N-continuous iff $f^{-1}(D)$ is N-open in U_1, ∀ N-open set D in U_2.[15]
2. N-δ-continuous iff $f^{-1}(D)$ is N-δ-open in U_1, ∀ N-open set D in U_2. [10]
3. N-Reg-continuous iff $f^{-1}(D)$ is N-Reg-open in U_1, ∀ N-open set D in U_2.[5]
4. N-α-continuous iff $f^{-1}(D)$ is N-α-open in U_1, ∀ N-open set D in U_2.[16]
5. N-pre-continuous iff $f^{-1}(D)$ is N-pre-open in U_1, ∀ N-open set D in U_2.[12]
6. N-semi-continuous iff $f^{-1}(D)$ is N-semi-open in U_1, ∀ N-open set D in U_2.[17]
7. N-β-continuous iff $f^{-1}(D)$ is N-β-open in U_1, ∀ N-open set D in U_2.[7]

Definitions 4.2:[10]

Let (U_1, $T_R(x)$) and (U_2, $T_R(Y)$) be two Nano topological space. Then, a mapping f: (U_1, $T_R(x)$) →(U_2, $T_R(Y)$) is said to be:

1. N-δ-continuous iff $f^{-1}(D)$ is N-δ-open in U_1, ∀ N-open set D in U_2.
2. N-δ-Reg-continuous iff $f^{-1}(D)$ is N-δ-Reg-open in U_1, ∀ N-open set D in U_2.
3. N-δ-α-continuous iff $f^{-1}(D)$ is N-δ-α-open in U_1,∀ N-open set D in U_2.
4. N-δ-pre-continuous iff $f^{-1}(D)$ is N-δ-pre-open in U_1, ∀ N-open set D in U_2.
5. N-δ-semi-continuous iff $f^{-1}(D)$ is N-δ-semi-open in U_1, ∀ N-open set D in U_2.
6. N-δ-β-continuous iff $f^{-1}(D)$ is N-δ-β-open in U_1,∀ N-open set D in U_2.

Definitions 4.3:

Let (U_1, $T_R(x)$) and (U_2, $T_R(Y)$) be two Nano topological space. Then, a mapping f: (U_1, $T_R(x)$) →(U_2, $T_R(Y)$) is said to be:

1. N-g-continuous iff $f^{-1}(D)$ is N-g-open in U_1, ∀ N-open set D in U_2.[2]
2. N-g-α-continuous iff $f^{-1}(D)$ is N-g-α-open in U_1, ∀ N-α-open set D in U_2.[18]
3. N-g-pre-continuous iff $f^{-1}(D)$ is N-g-pre-open in U_1, ∀ N-pre-open set D in U_2.[2]
4. N-g-semi-continuous iff $f^{-1}(D)$ is N-g-semi-open in U_1, ∀ N-semi-open set D in U_2.[2]
5. N-g -β-continuous iff $f^{-1}(D)$ is N-g-β-open in U_1, ∀ N-β-open set D in U_2.[14]

Definition 4.4:

Let (U_1, $T_R(x)$) and (U_2, $T_R(Y)$) be two Nano topological space. Then, a mapping f: (U_1, $T_R(x)$) →(U_2, $T_R(Y)$) is said to be N-g-δ-continuous iff $f^{-1}(D)$ is N-g-δ-open in U_1, ∀ N-δ-open set D in U_2.

Remark 4.5:

The next diagram explain the relation between N-g-continuous functions.

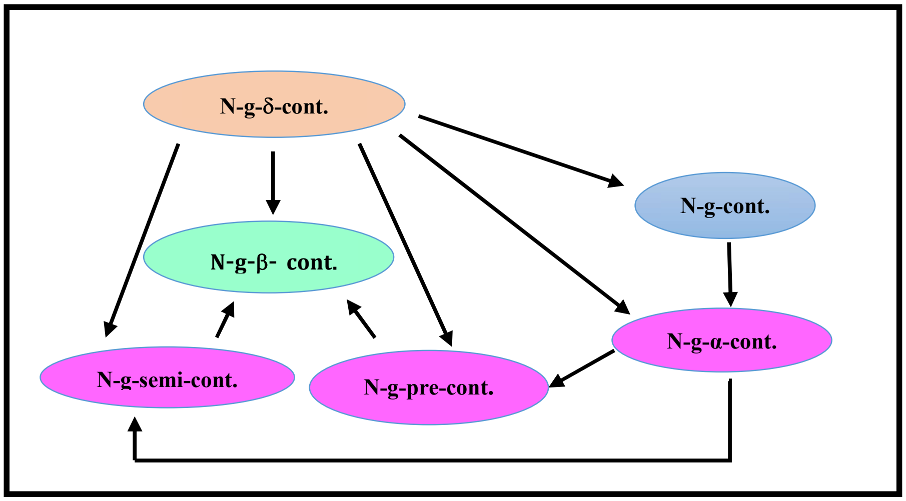

Diagram 4.1: the relation between N-g-continuous function.

Example 4.6:

Let U_1 = {k, l, m}, U_1/R = {{k, l}, {m}} and X = {k, m}. then, $T_R(x)$ = {U_1, ∅, {m}, {k, l}}, and let U_2 = {r, p, q} with U_2/R = {{r,p}, {q}} and D = {p, q}. Then, $T_R(D)$ = {U_2, ∅, {r, p}, {q}}.

Define f: $U_1 \to U_2$ as f(k) = r, f(l) = p, f(m) = q , f^{-1}(r, p) = {k, l) and $f^{-1}(U_1, ∅) = (U_2, ∅)$ then f is N-g-δ-continuous function.

Theorem 4.7:

f: (U_1, $T_R(x)$) →(U_2, $T_R(D)$) is N-g-δ-continuous iff f^{-1} (w) is N-g-δ-closed set in U_2, is N-g-δ-closed set in U_1.

Proof:

Let f is N-g-δ-continuous and w be N-g-δ-closed set in U_2. Then, U_2-w is N-g-δ-open set in U_2. Since f is N-g-δ-continuous, $f^{-1}(U_2\text{-}w)$ is N-g-δ-open set in U_1. Then, $U_1\text{-}f^{-1}(w)$ is N-g-δ-open set in U_1. Therefore, f^{-1}(w) is N-g-δ-closed set in U_1. Thus, f^{-1} (N-g-δ-closed) set in U_2 be N-g-δ-closed set in U_1.

Conversely,

Let w_1 be is N-g-δ-open set in U_2, then $U_2\text{-}w_1$ is N-g-δ-closed set in U_2. Then, $f^{-1}(U_2\text{-}w_1)$ is N-g-δ-closed set in U_1.Therefore, $f^{-1}(w_1)$ is N-g-δ-open set in U_1. Thus, f^{-1} (N-g-δ-open) set in U_2 is N-g-δ-open set in U_1. That is the function is N-g-δ-continuous.

Theorem 4.8:

f: (U_1, T_R(x)) →(U_2, T_R(D)) is N-g-δ-continuous iff N-g-clδ(f^{-1} (w)) ⊆ f^{-1}(N-g-clδ(w)) ∀ w ⊆U_2.

Proof:

Let f be N-g-δ-continuous and w ⊆ U_2. Then, cl(w) is N-g-δ-closed set in (U_2, T_R(D)). Therefore, f^{-1}(N-g-clδ(w)) is N-g-δ-closed set in (U_1, T_R(x)). That is f^{-1}(N-g-clδ(w)) = N-g-clδ[f^{-1}(N-g-clδ(w))]and N-g-clδ(w) ⊆ w. Then, f^{-1}(N-g-clδ(w) ⊆ f^{-1}(w). Therefore, N-g-clδ[f^{-1}(N-g-clδ(w))]⊆ N-g-clδ(f^{-1} (w)). That is f^{-1}(N-g-clδ(w))⊆ N-g-clδ(f^{-1} (w)).

Conversely,

If f^{-1}(N-g-clδ(w))⊆ N-g-clδ(f^{-1} (w)). Then, if w is N-g-closed set in U_2, N-g-clδ(w) = w. also, f^{-1}(N-g-clδ(w)) ⊆ N-g-clδ(f^{-1} (w)). That is f^{-1} (w) ⊆ N-g-clδ(f^{-1} (w)). But N-g-clδ(f^{-1}(w)) ⊆ f^{-1}(w). Therefore, f^{-1}(w) = N-g-clδ(f^{-1} (w)), thus f^{-1} (w) is N-g-closed set in U_1, for all N-g-closed set w in U_2. Therefore, f is N-g-δ-continuous function.

Definition 4.9:

A function f: (U_1, T_R(x)) →(U_2, T_R(D)) is called N-g-δ-totally continuous if the invers image ∀ N-g-δ-open set in U_2 is N-g-δ-clopen set in U_1.

Example 4.10:

Let U_1 = {k, l, m} with U_1/R = {{k{, {l, m}} and X = {k, l} then T_R(x) = {∅, U_1, {k}, {l, m}} are N-g-δ-clopen sets in U_1.

And let U_2 = {1, 2, 3} with U_2/R = {{1}, {2, 3}}

D = {1, 2} then, T_R(D) = {U_2, ∅, {1}, {2,3}} is N-g-δ-open set in U_2.

Define f: U_1→ U_2 as f(k) = 1 , f(l) = 2 and f(m) = 3. Then, f^{-1}(2, 3) = {l, m} ,f^{-1}(1) = k , f^{-1}(U_2, ∅) = (U_1, ∅). Then, the function is N-g-δ-totally continuous.

Remark 4.11:

A function f: (U_1, T_R(x)) →(U_2, T_R(D)) is called N-g-δ-totally continuous iff f^{-1}(N-g-δ-closed set) in U_2 is N-g-δ-clopen set in U_1.

Theorem 4.12:

If f: (U_1, T_R(x)) →(U_2, T_R(D)) be a N-g-δ-continuous mapping if U_R(x) = U_1 and L_R(x) ≠ ∅. Then, f is N-g-δ-totally continuous.

Proof:

Let $U_R(x) = U_1$, $L_R(x) \neq \emptyset$ and f be N-g-δ-continuous function then $T_R(x) = \{ U_1, \emptyset, L_R(x), B_R(x)\}$ and $T_R^c(x) = \{ \emptyset, U_1, \overline{L_R(x)}, \overline{B_R(x)}\}$ since $U_R(x) = U$ that mean $L_R(x) = \overline{B_R(x)}$, $\overline{L_R(x)} = B_R(x)$, since f is N-g-δ-continuous. So that the invers image ∀ N-g-δ-open set in U_2 is N-g-δ-open set in U_1, as ∀ N-g-δ-open set in U_2 is N-g-δ-closed set in U_1. Hence f is N-g-δ-totally continuous.

Theorem 4.13:

The composition of two N-g-δ-totally continuous map is N-g-δ-totally continuous.

Proof:

Let f: $(U_1, T_R(x)) \to (U_2, T_R(D))$ and g: $(U_2, T_R(D)) \to (U_3, T_R(y))$ be two N-g-δtotally continuous functions. Let U_2 be N-g-δ-open set in U_3. Since g is N-g-δ-totally continuous. Then, $g^{-1}(U_2)$ is N-g-δ-clopen in $U_2 \to g^{-1}(U_2)$is N-g-δ-open in U_2. Since f is N-g-δ-totally continuous. Then $f^{-1}(g^{-1}(U_2))$ is N-g-δ-clopen in $U_1 \to (g \circ f)^{-1}$ is N-g-δ-clopen in $U_1 \to$ gof is N-g-δ-totally continuous.

Theorem 4.14:

Every N-g-δ-totally continuous function is N-g-δ--continuous function.

Proof:

Let f: $(U_1, T_R(x)) \to (U_2, T_R(D))$ is N-g-δ-totally continuous function and let F be any N-g-δ-open set in U_2. Since f is N-g-δ-totally continuous then $f^{-1}(F)$ is N-g-δ-clopen set in U_1. Then $f^{-1}(F)$ is N-g-δ-open set in U_1. Then, f is N-g-δ-continuous function.

Definition 4.15:

Let : $(U_1, T_R(x))$ and $(U_2, T_R(D))$ N-g-δ-topological spaces then, the map f: $U_1 \to U_2$ N-g-δ-contra continuous, if the inverse image ∀ N-g-δ-open set in U_2 is N-g-δ-closed set in U_1.

Example 4.16:

Let $U_1 = \{k, l, m, n\}$ with $U_1/R = \{\{k\}, \{l\}, \{m, n\}\}$ and $X=\{k, l, m\}$ then $T_R(x) = \{ U_1, \emptyset, \{k, l\}, \{m, n\}\}$ and let $U_2 = \{a, b, c, d\}$ with $U_2/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $H = \{a, b\}$ then $T_R(H) = \{U_2, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$

Define f : $U_1 \to U_2$ as f(k) = a , f(l) = b, f(m) = c , f(n) = d. then, $f^{-1}(\{a\}) = \{k\}$, $f^{-1}(\{b, c\}) = \{l, m\}$ $f^{-1}(\{a, b, c\}) = \{ k, l, m\}$ and $f^{-1}(\emptyset, U_2) = \{\emptyset, U_1\}$ then f is N-g-δ-contra continuous.

Remark 4.17:

If N-g-δ-continuous function not be necessary N-g-δ-contra continuous. We can show that by the following example.

Example 4.18:

Let $U_1 = \{1, 2, 3, 4\}$ with $U_1/R = \{\{1\}, \{2,4\}, \{3\}\}$ and $X=\{1, 2\}$ then $T_R(x) = \{ U_1 , \emptyset, \{1\}, \{2, 4\}, \{1, 2, 4\}\}$ and let $U_2 = \{a, b, c, d\}$ with $U_2/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $D = \{a, b\}$ then $T_R(D) = \{U_2, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$

Define $f : (U_1, T_R(x)) \to (U_2, T_R(D))$ as $f(1) = a$, $f(2) = b$, $f(3) = d$, $f(4) = c$. then, $f^{-1}(\{a\}) = \{1\}$, $f^{-1}(\{b, c\}) = \{2, 4\}$ $f^{-1}(\{a, b, c\}) = \{ 1, 2, 4\}$ are not N-g-δ-closed sets. Then, f is N-g-δ-continuous function but not N-g-δ-contra continuous.

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