

**NUMERICAL INVESTIGATION OF MHD HYBRID CU – AL<sub>2</sub>O<sub>3</sub>/WATER  
NANOFLUID WITH THERMAL RADIATION**

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**ABSTRACT:**

The current study looks at the 2D MHD flow of Hybrid Cu – Al<sub>2</sub>O<sub>3</sub> / H<sub>2</sub>O-based nanofluid caused by a porous media stretching sheet with thermal radiation. Hybrid nanoparticles of copper (Cu) and alumina (Al<sub>2</sub>O<sub>3</sub>) are considered for investigation, along with H<sub>2</sub>O as the base fluid. After a suitable similarity transformation, the governing equations are then rebuilt as a collection of non-linear ODE. To solve the equations, we used the fourth order Runge Kutta numerical technique. The influence of various parameters on the velocity and temperature profiles is graphically depicted. Based on our observations, we discovered that raising the magnetic and radiation parameters thickens the thermal boundary layer. It is also discovered that by using a magnetic field, porous media, and varying the volume % of the nanoparticles, the speed of the hybrid nanofluid may be altered.

**Keywords :** Thermal Radiation, Heat Transfer, Mass Transfer, Hybrid Nanofluid, Numerical Results

**INTRODUCTION:**

Various novel strategies were utilised in earlier decades to improve heat transfer rates and attain varying levels of thermal efficiency. Improving thermal conductivity is critical to accomplishing this. Finally, to improve thermal conductivity, various efforts were made to disperse high thermal conducting solid particles into liquids, and nanofluids developed in the early 1990s. Many studies in nanofluids have been conducted to meet the needs of industrial applications. The advantages of "hybrid nanofluid" include increased thermal conductivity, enhanced heat transfer, stability, the benefits and drawbacks of individual suspensions. Nanofluids' outstanding thermal conductivity leads into improved energy efficiency.

Choi [5] was the first to notice that thermal conductivity may be increased by nanoparticles dispersed in base fluid, improving the fluid's heat transfer rate. As a result of the discovery, Nield and Kuznetsov [8] applied model of Buongiorno to the boundary layer problem. Niihara [9] demonstrated nanocomposites with improved mechanical and thermal properties through the use of a new material design idea. Suresh et al. [12] conducted research to create a hybrid

(Al<sub>2</sub>O<sub>3</sub> /Cu=water) nanofluid. They explained nanocomposites with a new nanomaterial design concept that significantly improved thermal and mechanical properties. For enhancing heat transfer Suriyauma Devi et.al [11] used the hybrid nanofluid and compared their result with existing nanofluid. Nur Syahirah Wahid et.al [7] explored the analytical way of approach for enhancing heat transfer using hybrid nanofluid. Using an analytical method, Rajesh et al. [10] examined the heat transfer characteristics of hybrid unstable nanofluid flow across an infinite vertical plate. Jabeeb et al. [6] used computational software and various standard techniques to solve the steady, incompressible problems of MHD flow over a nonlinear stretching sheet. Sheob Rashid et al. [13] used a differential transformation technique to solve a free convection MHD slip flow problem across a stretched sheet radially with thermal radiation. Using the two-phase mixture model, Simon et al [14] investigate the numerical analysis of Al<sub>2</sub>O<sub>3</sub>-Water nanofluid. The effect of heat radiation on a transient MHD flow with MT through a fixed vertical plate was explored by Ahmed and Sarmah[3]. Alabraba et al. [2] explored the interaction of free convection and heat radiation through permeable medium with solet and dufour effects. Alagoa et al.[1] examined the effect of radiation MHD flow with time dependent suction. Cess [4] investigated the relationship between thermal radiation and free convection heat transfer.

## MATHEMATICAL FORMULATION

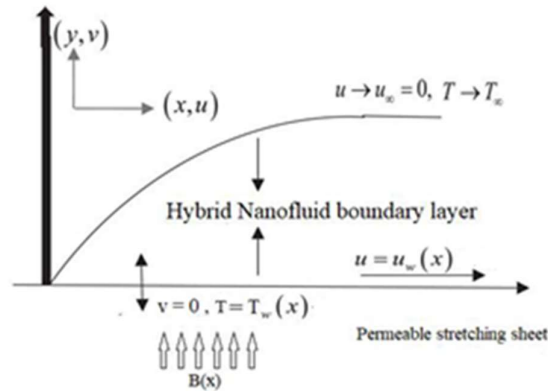


Figure-1

Figure 1 displays the continuous flow and heat transmission of a Cu- Al<sub>2</sub>O<sub>3</sub>/ H<sub>2</sub>O -based hybrid nanofluid with permeable stretched surface. The x-axis is parallel to the sheet, whereas the y-axis is measured perpendicular to the sheet. A magnetic field B(x) is applied in the direction of the y-axis.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \frac{\mu_{hnf} \partial^2 u}{\rho_{hnf} \partial y^2} - \frac{\sigma_{hnf}}{\rho_{hnf}} B^2 u - \frac{\mu_{hnf} u}{\rho_{hnf} k_o} \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa_{hnf} \partial^2 T}{(\rho C_p)_{hnf} \partial y^2} + \frac{\mu_{hnf}}{(\rho C_p)_{hnf}} \left( \frac{\partial u}{\partial y} \right)^2 - \frac{\sigma_{hnf} B^2 u^2}{(\rho C_p)_{hnf}} - \frac{1}{(\rho C_p)_{hnf}} \frac{\partial q_r}{\partial y} \quad (3)$$

with boundary conditions,

$$\begin{aligned} u(x, 0) &= cx, \\ v(x, 0) &= 0, \end{aligned}$$

$$\begin{aligned} T(x, 0) &= T_w, \\ u(x, \infty) &= 0, \\ T(x, \infty) &= T_\infty \end{aligned} \tag{4}$$

The Rosseland approximation for radiative heat flux is,

$$\frac{\partial q_r}{\partial y} = \frac{-4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}$$

We may obtain by generalising the Taylor series and disregarding the higher order terms.

$$T^4 = 4T_\infty^3 T - 3T_\infty^4$$

Hence above equation becomes,

$$\frac{\partial q_r}{\partial y} = \frac{-16\sigma^*}{3k^*} T_\infty^3 \frac{\partial^2 T}{\partial y^2}$$

Considering the similarity transformation,

$$\begin{aligned} u &= cx f'(\eta); \\ v &= -\sqrt{cv_f} f(\eta); \\ \eta &= \sqrt{\frac{c}{v_f}} y; \\ \theta(\eta) &= \frac{T - T_\infty}{T_w - T_\infty} \end{aligned}$$

The current study introduces a new type of thermophysical property to analyse the boundary layer for Hybrid Nanofluid. To make the required Hybrid nanofluid, Cu nanoparticles are combined with 0.1 vol. of Al2O3/water. In this model, an Al2O3 nanoparticle ( $\phi_1$ ) is introduced to the base fluid with a 0.1 vol. solid volume fraction (i.e.,  $\phi_1 = 0.1$ , which remains constant throughout the problem), and then Cu ( $\phi_2$ ) is added with varying solid volume fractions to generate the Cu-Al2O3/Water hybrid nanofluid.

**Table :1 Thermophysical properties of water and nanoparticles**

PROPERTIES	Al <sub>2</sub> O <sub>3</sub>	Cu	WATER(f)
$\rho C_p$	3970	8933	997.0
$C_p$	765	385	4180
$k$	40	400	0.6071

**Table : 2 Properties of Nanofluid and Hybrid Nanofluid**

$\Phi$		$C_p$	$k$	$\rho$	$\mu$
0.00	Water	4182	0.597	998.2	$998 \times 10^{-6}$
0.01	Al <sub>2</sub> O <sub>3</sub>	4050	0.614	1027.9	$1080 \times 10^{-6}$
0.01	Cu-Al <sub>2</sub> O <sub>3</sub>	4076	0.657	1029.8	$1602 \times 10^{-6}$

**Table : 3**  
**Thermophysical properties model for hybrid nanofluid:**

PROPERTIES	HYBRID NANOFLUID	NANOFLUID
Density	$\rho_{hnf} = \{(1-\varphi_2)[(1 - \varphi_1)\rho_f + \varphi_1\rho_{s1}]\} + \varphi_2\rho_{s2}$	$\rho_{nf} = (1 - \varphi)\rho_f + \varphi\rho_s$
Heat Capacity	$(\rho C_p)_{hnf} = \{(1-\varphi_2)[(1 - \varphi_1)(C_p\rho)_f + \varphi_1(\rho C_p)_{s1}]\} + \varphi_2(C_p\rho)_{s2}$	$(\rho C_p)_{nf} = (1 - \varphi)(\rho C_p)_f + \varphi(\rho C_p)_s$
Viscosity	$\mu_{hnf} = \frac{\mu_f}{(1 - \varphi_1)^{2.5}(1 - \varphi_2)^{2.5}}$	$\mu_{nf} = \frac{\mu_f}{(1 - \varphi)^{2.5}}$
Thermal Conductivity	$\frac{k_{hnf}}{k_{bf}} = \frac{k_{s2} + (n - 1)k_{bf} - (n - 1)\varphi_2(k_{bf} - k_{s2})}{k_{s2} + (n - 1)k_{bf} + \varphi_2(k_{bf} - k_{s2})}$ <p>Where,</p> $\frac{k_{bf}}{k_f} = \frac{k_{s1} + (n - 1)k_f - (n - 1)\varphi_1(k_f - k_{s1})}{k_{s1} + (n - 1)k_f + \varphi_1(k_f - k_{s1})}$	$\frac{k_{nf}}{k_f} = \frac{k_s + (n - 1)k_f - (n - 1)\varphi(k_f - k_s)}{k_s + (n - 1)k_f + \varphi(k_f - k_s)}$

Substituting the similarity transformation in the governing equation, we get

$$f'''' + \frac{B_1}{B_2} f f'' - \frac{B_1}{B_2} f'^2 - \frac{MB_3}{B_2} f' - K_p f' = 0$$

$$\frac{\theta''}{Pr} \left( B_4 + \frac{4}{3} R \right) + B_5 f \theta' + B_2 E c f''^2 + B_3 E c M f'^2 = 0$$

with the boundary conditions,

$$f(0) = 0, f'(0) = 1, \theta(0) = 1, f'(\infty) = 0, \theta(\infty) = 0$$

The flow parameters in this investigation are as follows:

$$B_1 = \frac{\rho_{hnf}}{\rho_f}; \quad B_2 = \frac{\mu_{hnf}}{\mu_f}; \quad B_3 = \frac{(\rho C_p)_{hnf}}{(\rho C_p)_f}; \quad B_4 = \frac{k_{hnf}}{k_f} \quad B_5 = \frac{\sigma_{hnf}}{\sigma_f}$$

$$M = \frac{\sigma_f B_0^2}{c \rho_f}; \quad K_p = \frac{\nu_f}{c k_0}; \quad Pr = \frac{\nu_f (\rho C_p)_f}{k_f}; \quad R = \frac{4 T_\infty^3 \sigma^*}{\kappa k^*}; \quad Ec = \frac{c^2 \chi^2}{C_p (T_w - T_\infty)}$$

**RESULTS AND DISCUSSION:**

Figure 2 depicts the changes in velocity of a hybrid nanofluid for different values of M. As the magnetic parameter increases, the velocity decreases. When the strength of the magnetic field increases, a retarding force known as the Lorentz force is produced, forcing the flow to slow down. The velocity profile decreases as the magnetic field increases. As seen in Fig. (3), the temperature range experienced inside the boundary layer increases as M grows. The increased

Lorentz force impact increases the flow's frictional resistance, raising the temperature in the boundary layer region. As a result, increasing  $M$  raises the temperature.

Figures (4) and (5) show how  $K_p$  affects the velocity and temperature profiles respectively. The figures show that the velocity of the Cu - Al<sub>2</sub>O<sub>3</sub>/water hybrid nanofluid drops as the value of  $K_p$  increases. This happens because an increase in  $K_p$  amplifies the porous layer, causing velocity and temperature in the boundary layer to decrease. According to the graph, the hybrid nanofluid performs better in terms of controlling velocity and maintaining temperature.

The effect of  $Pr$  on dimensionless temperature for hybrid nanofluid is seen in Fig. (6). As  $Pr$  increases, the temperature within the boundary layer area decreases. The momentum-to-temperature diffusivity ratio is defined as  $Pr$ . Because momentum diffuses faster than heat, a high Prandtl value suggests poor thermal conductivity and a thinner thermal layer structure. The heat transfer rate of the fluid increases as the Prandtl number increases, causing the temperature of the boundary layer to decrease. Figures (7) and (8) depict the effect of Eckert number( $Ec$ ) and radiation parameter ( $R$ ) on the temperature profile. The temperature obviously rises as the radiation parameter and Eckert number rise. This is due to the effect of heat energy being released into the fluid when thermal radiation increases.

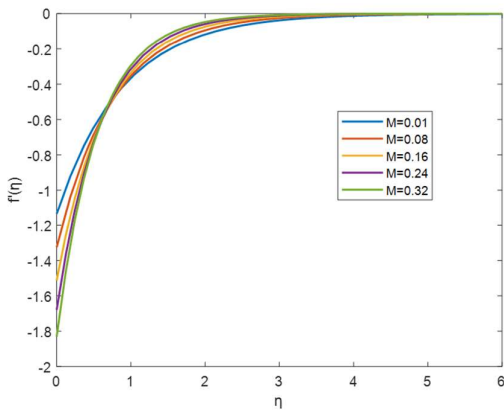


Fig 2 Velocity distribution for various  $M$  values

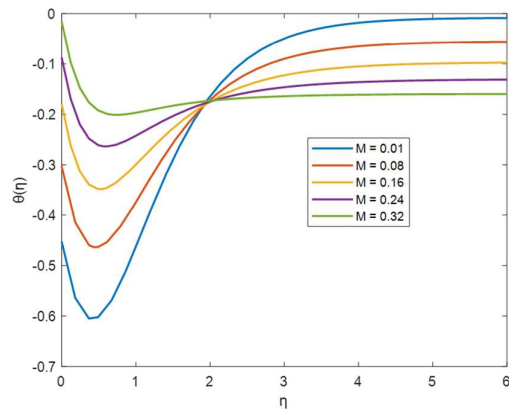


Fig 3 Temperature distribution for various  $M$  values

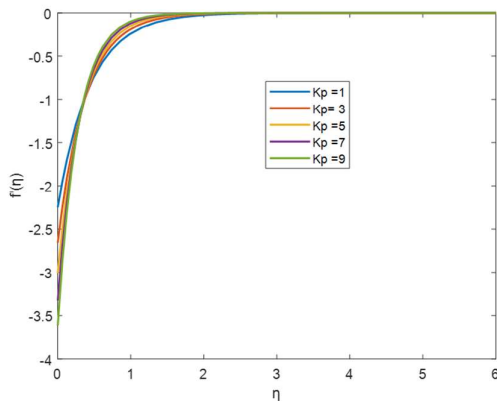


Fig 4 Velocity distribution for various  $K_p$  values

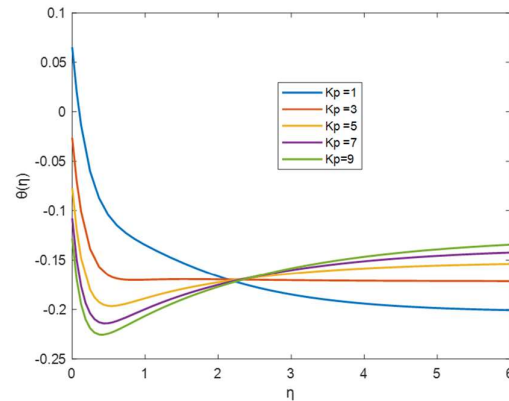


Fig 5 Temperature distribution for various  $K_p$  values

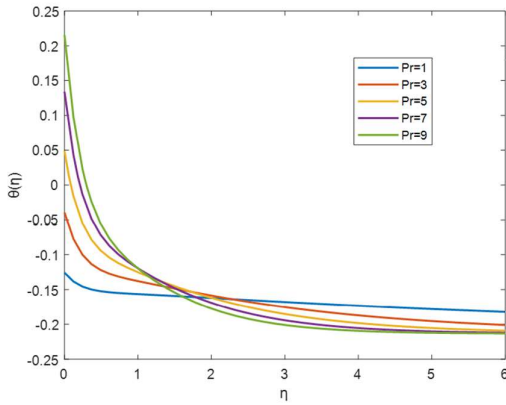


Fig 6 Temperature distribution for various Pr values

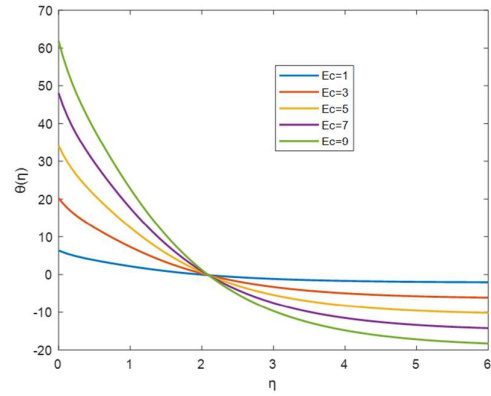


Fig 7 Temperature distribution for various Ec values

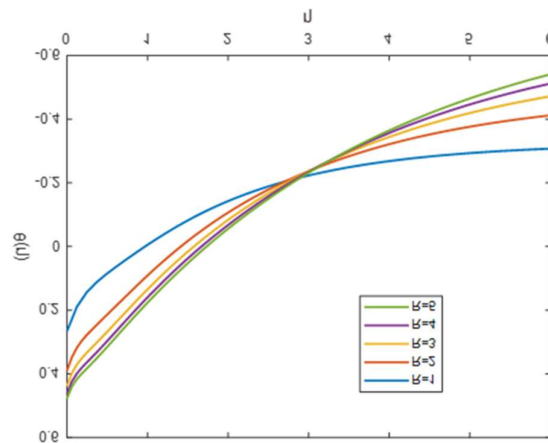


Fig 8 Temperature distribution for various R values

### CONCLUSION

The current work focused on hybrid nanofluid flow across a stretching sheet. A parametric research was conducted in order to have a clear physical understanding of the situation. The inquiry makes use of a novel fluid called Hybrid nanofluid (Cu - Al<sub>2</sub>O<sub>3</sub>). The key benefit of employing this next generation Hybrid nanofluid is that it enhances thermal properties. The study's conclusion is as follows:

- When the magnetic field is increased, the velocity of the hybrid nanofluid flow inside the boundary layer can decrease.
- When the magnetic field, Eckert number and thermal radiation grow, the temperature of the hybrid nanofluid rises.
- When Kp increases, the velocity and temperature decrease.
- When the Prandtl number increases, the temperature of the hybrid nanofluid falls.

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