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CALCULATION OF ELECTROMAGNETIC FORCE BY THE MAXWELL TENSOR AND VIRTUAL WORK IN THE FINITE ELEMENT METHOD

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ABSTRACT

The main goal of this work is to investigation of the electromagnetic force in the electromagnetic by two methods, Virtual Work Principle and the Maxwell Tensor magnetic pressure, through fine elements method the results are compared with theoretical approach.

Keywords: electromagnetic force, electromagnetic, Maxwell stress tensor, virtual work

INTRODUCTION

The calculation of the electromagnetic force in the electromagnet is important to find the static voltage curve, the latter is the first step to reach a dynamic operational study of the circuit breakers, and it is very important in the case of small-sized circuit breakers, where their dynamics are very related to the electrical supply.

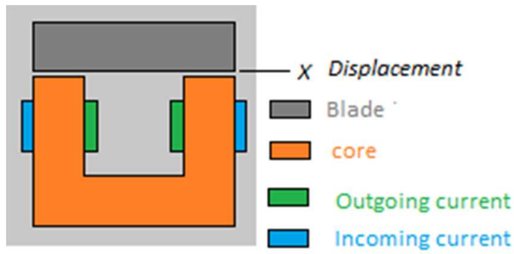
There are several methods for calculating this force, the most famous of which is the Maxwell stress tensor, which is especially useful for finding electromechanical boundary conditions in summary form. And find the total electromagnetic force acting on the body, and the method of virtual works that depends on the imposition of an apparent movement and we write the energy balance in the sense of obtaining the value of the energy difference due to the virtual displacement. Concerning the local force calculation, one has the force density $Id\vec{l} \times \vec{B}$ (Lorentz formula) in conductors, and $-\frac{1}{2}\left(H_t^2\nabla\mu - B_n^2\nabla\nu\right)$ in magnetized materials (without magnetostriction).

In this study, we examine the findings of these methods by comparing them to theoretical electromagnet results. The finite element approach is utilized to solve the problem, accruing to its great numerical yield and flexibility (accuracy required).

ELECTROMAGNET GEOMETRY

A good finite element analysis begins with a full description of the geometry of the system, so we first represent it in Fig. 1, which is a cross-section of an electromagnet, consisting of a core, blade and coils.

The core and blade are made from steel with high permeability and excited by a current flowing in a coil of N turns causing the blade to move upwards, the whole surrounded by an empty space. The position of the blade is defined by the parameter *X*, and the dimensions of the electromagnet are represented in Table(01).



Fig(01). Electromagnet geometry

TABLE1. DIMENSIONS OF THE ELECTROMAGNET

	x(mm)	y(mm)
Blade	80	20
Core outside	80	60
Core inside	40	40
Coil (outgoing or incoming current)	10	20

THEORETICAL APPROACH OF THE MAGNETIC FORCE

We must start by calculating the reluctance of the circuit with respect to displacement of the blade X , we will deduce the inductance L by the definition formula [1] -[2] :

$$L = \frac{n^2}{\Re} \qquad (1)$$

Assuming that the magnetic circuit contains only one closed loop path, where the flux is distributed in the core, the blade and the air-gab. Therefore, the reluctance become as follows:

$$\Re = \oint_l \frac{dl}{\mu S} \qquad (2)$$

we successively calculate both reluctance of magnetic circuit ,the blade and the core have same relative permeability, and reluctance of air-gab reluctance by below relationships:

$$\Re_{air\_gap} = \frac{X}{\mu_0 S}, \Re_{bc} = \frac{L}{\mu S} \qquad (3)$$

Then we apply the following formula to calculate the electromagnetic force which to exerted on the blade

$$F_{constant\_supply} = \frac{1}{2}\frac{dL}{dX} I^2 \qquad (4)$$

NUMERICAL APPROXIMATION

To find the overall system variables, such as internal energy and force, we solve the partial differential equations that govern each region of the electromagnet of the following form:

$$\sigma \frac{\partial \vec{A}}{\partial t} + \nabla \times \frac{1}{\mu} \nabla \times \vec{A} = \vec{J}_e \qquad (5)$$

We use the commonly used method because of its flexibility and high numerical yield (accuracy required), which is the finite element method.
By calculating the raw result of the magnetic potential
We can compute the magnetic energy of the system, as well as the components of the Maxwell tensor.

THE PRINCIPLE OF MAXWELL STRESS TENSOR

In electromagnetic devices, several tensors of electromagnetic constraints, including that of Maxwell, are commonly used for the calculation of the global force of magnetic origin which is exercised on Mobile parts such as the nucleus of an electromagnetic or the rotor of an electric motor. [3-6].
The electromagnetic force ***F*** is obtained by using the Maxwell stress tensor[7] :

$$F_v = \iiint_v \nabla T dv \qquad (6)$$

by the application of the divergence law, the equation 6 becomes :

$$F_s = \oint_s T.nds \qquad (7)$$

In terms of components, the stress tensor T on the outer surface of the blade [8]:

$$\mathrm{T} = [\mathrm{T}_{ij}], \mathrm{T}_{ij} = \frac{1}{\mu_0}\left(B_i B_j - \frac{1}{2}\delta_{ij} B^2\right), (i, j = x, y, z)$$

$$\mathrm{T} = \frac{1}{\mu_0}\begin{bmatrix} B_x^2 - \frac{1}{2}B^2 & B_x B_y & B_x B_z \\ B_y B_x & B_x^2 - \frac{1}{2}B^2 & B_y B_z \\ B_z B_x & B_z B_y & B_x^2 - \frac{1}{2}B^2 \end{bmatrix} (8)$$

In equation (8) the quantities *Bx*, *By* and *Bz* are magnetic flux density components and *B* its module in the element.
We can also write:

$$F = -\frac{1}{2}\left(H_t^2 \nabla \mu - B_n^2 \nabla v\right) \qquad (9)$$

Here it is worth noting that this method cannot be used when the blade is attached to the carcass. Because in this case, the gradient of both permeability μ and her inverse υ is equal to zero. After simplifying the previous equations, to calculate the magnetic force, we apply the equation below :

$$F = \frac{1}{2\mu_0} \int_{\Omega} (-(B_x^2 - B_y^2)n_y + 2B_x B_y n_x) dl \quad (10)$$

in vector form:

$$\vec{F} = \begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = \frac{1}{2} \begin{Bmatrix} (H_x B_x + H_y B_y) \times n_x + (n_x H_x + n_y H_y) \times B_x \\ (H_x B_x + H_y B_y) \times n_y + (n_x H_x + n_y H_y) \times B_y \end{Bmatrix} (11)$$

For our case, as shown in Fig. 1. And Fig. 5, the force has only one component in the direction (OY) thence the previous equation can be simplified to:

‘

$$F = -\frac{1}{2\mu_0} \oint_s B_y n_y ds \qquad (12)$$

THE PRINCIPLE OF VIRTUAL WORK

The relation between the force exerted on the blade is the variation of the energy of the system with respect to the blade position. It is given as follows [9-10]:

$$F = \frac{dw}{dX} \qquad (13)$$

Where:

$$\frac{dw}{dX} = \frac{1}{2} \vec{B}.\vec{H} \qquad (14)$$

When the relative permeability μ_r of the materials is constant, i.e. practically in the linear region of the curve, the vectors $\vec{B}$ and $\vec{H}$ are parallel, and then we can express the energy density in the following form:

$$\frac{dw}{dX} = \frac{1}{2} \frac{B^2}{\mu_0 \mu_r} \qquad (15)$$

We must calculate the total energy of the system in the current situation, and then we calculate the energy after moving the blade and calculate the difference and divide it by the amount of movement, during the simulation we move the blade by a fixed amount equal to 0.25 *mm*.

DYNAMIC STUDY OF ELECTROMAGNET

In electrical equilibrium, we have a relationship between supply and current in the coil and inductance, the latter of which changes in terms of the position of the blade. since we work in the linear regime [12-13]:

$$V = rI + L(X)\frac{\partial I}{\partial t} + I\frac{\partial X}{\partial t}\frac{\partial L(X)}{\partial X} \qquad (16)$$

And in the mechanic equilibrium we have:

$$\frac{\partial^2 X}{\partial t^2} = \frac{1}{2m}\frac{\partial L(X)}{\partial X}I^2 + g \qquad (17)$$

RESULTS AND DISCUSSION

We will now display the magnetic force arrows on the circumference of the board (the blade). This result is valid only if the permeability of the material is much greater compared to the permeability of the surrounding material, and this is our case.

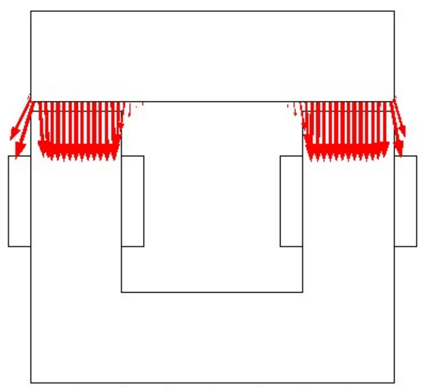


Fig. 1. The intensity of the force induced in the blade

to verify that the problem is indeed pose, first of all we see which the equipotential lines loop back and guide through the core, the air gap and the blade as shown in Fig. 2..

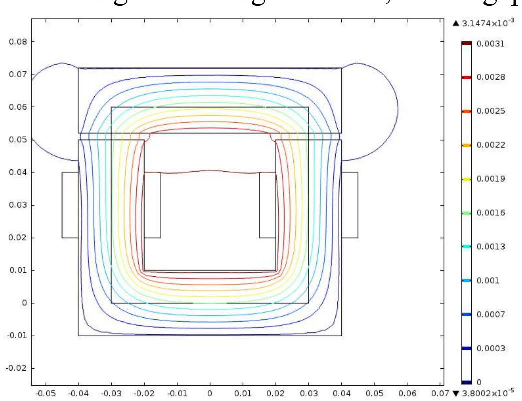


Fig. 2. Equipotential lines (A_z)

Then the representation of the magnetic flux density in color gradations makes it possible to highlight the saturation zones as shown in Fig. 3., which shows that the internal corners of the core are saturated. It is important that we use a sufficiently refined mesh to ensure adequate results as shows Fig. 3. for mesh1 and Fig. 4. for mesh2.

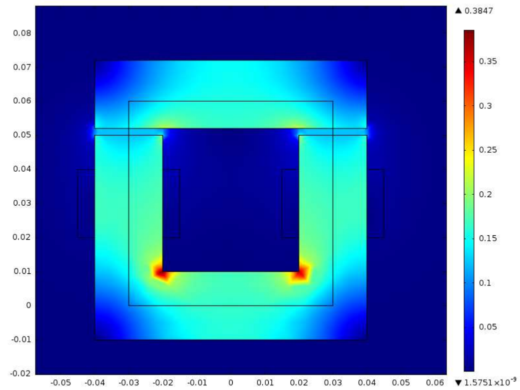


Fig. 3. Magnetic flux density for mesh1

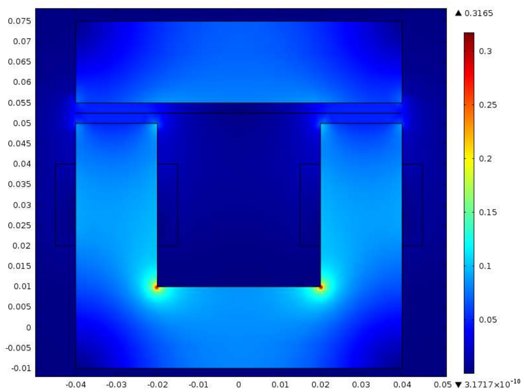


Fig. 4. Magnetic flux density for mesh2

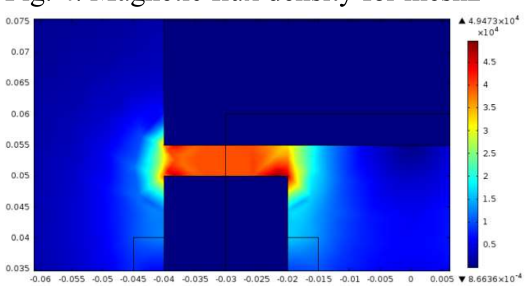


Fig. 5. Magnetic field in air-gab

now we apply a current density in the coils equal , and we vary the position of the blade by X, i.e. if X increases the blade is brought closer to the core.

We can also verify the Ampere's law by integrating the magnetic field along the average line AB in the Fig. 5.:

We have:

$$\oint_{l=L+2X} \vec{H}.d\vec{l} = 400.26A \qquad (18)$$

$$\oint_{l=2X} \vec{H}.d\vec{l} = 394.9A \qquad (19)$$

And:

$$\iint_{s} \vec{J}_e.d\vec{s} = 2\times 200A = 400A \qquad (20)$$

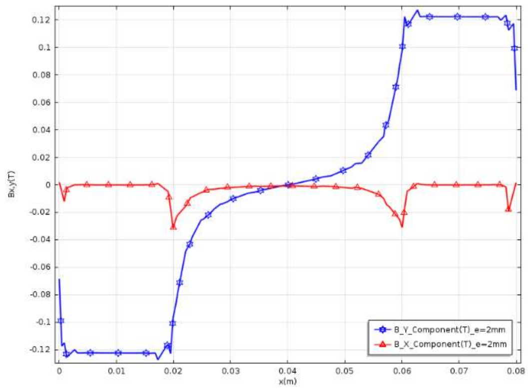


Fig. 6. Magnetic induction components B_x and B_y in air-gab

To calculate the force applied to the blade, we simplify Equation 2 to become of the form

$$F = \frac{-\mu S}{\left(2X\mu_r + L\right)^2} I^2 \qquad (21)$$

it is clearly that this electromagnetic force which is exerted on the blade is on the one hand proportional to the square of the excitation current, and on the other hand almost inversely proportional to the square of the distance X which separates the blade at the carcass.
After simplifying equation 5, we must mention that the force is the derivative of energy with respect to distance X which separating the blade and the core under a constant current supply, we find the value of the energy stored in the system (the air gap) as follows
The numerical results are in Fig 7. we can note that the numerical solutions of the two methods, Maxwell tensor and Virtual work, as well as the exact solution produce almost the same results.

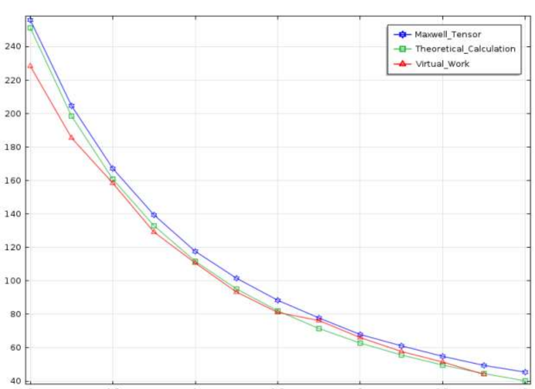


Fig. 7. Force in Newton as a function of position of Blade by three methods

CONCLUSION

In this paper, we have calculated the magnetic force resulting from the electromagnetic using the Maxwell tensor method, after having simplified it and adapting it to our case, and the virtual work method then we compared the results with values theoretical to show us that the results obtained are very close.

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